Journal of Mathematical Sociology
2001, Vol. 25(1), pp. 43–67
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PRE-TRANSITIVE BALANCE MECHANISMS FOR SIGNED NETWORKS*

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(Received April 14, 2000; In final form October 10, 2000)

The fundamental empirical structural balance hypothesis claims that human signed networks tend towards balance over time. Balance theorists assume that there is a balance theoretic mechanism whose cumulative effect drives the evolution of signed social structures towards balance. In previous work, we used a line index of imbalance to measure the imbalance of a network through time. Consistent with balance theory, we found a steady movement towards balance in the well known "Newcomb data". The balance mechanisms were, at best, implicit in that earlier analysis and our use of the line index of imbalance meant that we ignored triples. Here, we consider triples with the simple hypothesis that balanced triples exist at all times and, that through time, the balanced triples become more frequent while the imbalanced triples become less frequent. We examine pre-transitive balance conditions defined in terms of the $(i \rightarrow j, j \rightarrow k)$ ties and count the frequencies of the completion tie $(i \rightarrow k)$ for each of the combinations of tie signs in the pre-transitive condition. The basic structural balance theoretic hypotheses are supported – but only partially. Worse, from a balance theoretic viewpoint, there are triples for which the fundamental structural balance hypothesis is *contradicted.* We construct three substantive arguments to account for the exceptions and end with a plea for the collection of much more appropriate data in order to disentangle multiple mechanisms.

*We appreciate the many excellent comments of Evelien Zeggelink and Frans Stokman, suggestions that helped us improve the presentation of our ideas greatly.
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1 INTRODUCTION AND A POINT OF DEPARTURE

Heider (1946) is credited with the first intuitive and general statement of balance theory. His pioneering efforts provided a foundation for much substantive and empirical work. These efforts can be collected under the label of "structural balance theories". Assessment of the worth of this cumulated work varies. Structural balance can be viewed as part of a substantive success story in the social sciences (Davis, 1979) or as a stagnated theory full of empty promises (Opp, 1984). To the extent that real problems were recognized, attempts to revive structural balance theory took a variety of forms ranging from detailed re-specifications of formal systems representing balance theories (for example, Nagasawa and Light, 1983; Alessio, 1990) to close examination of the varieties of balance – where the Heider (1946) version is recast with modifications (see, for example Newcomb, 1968) – to further experimental work (for example, Crano and Cooper, 1973).

Charmed by Davis' (1979, p. 52) characterization of a "nifty theory" as "one that is falsifiable, nonobvious, and simple" we have put the subtle details of the accumulated discussion to one side. We think that another feature of a nifty theory is that it is predictive through time patterns by which (some) social processes operate and social structures emerge. In our view, structural balance (and ranked-clusters models), regardless of their problems, remain nifty and fruitful. Here, we extend this characterization to include time explicitly. This additional temporal feature is dictated by a concern with through time mechanisms and the need to examine longitudinal data. To do this, we use again the "Newcomb data", as recorded and reported by Nordlie (1958) and discussed in Newcomb (1961). Our intent is to examine "pre-transitive" conditions as a part of a set of balance mechanisms by which signed networks change through time. More precisely, for a trio of actors i, j and k, we examine configurations of $i \rightarrow j$ and $j \rightarrow k$ where the ties are signed as the "pre-conditions" and examine the $i \rightarrow k$ ties to see if such triples are completed in ways that are consistent with balance theoretic ideas.

The data involve 17 actors who each ranked each other with regard to affect over the course of a 15 week semester. This sequence of 15 sociomatrices of ranked data present serious problems requiring some kind of recoding activity. We have examined these data before

(Doreian *et al.*, 1996) but take a different course here. In our earlier work, we examined reciprocity, transitivity, and balance in a sequence of distinct analyses. We found that the amount of reciprocity was significant[1] in the first week and oscillated slightly around that initial level throughout the study period.

For transitivity, we used Freeman's (1992) discussion of an ultrametric as a point of departure. Letting d_{ijt} be a distance measure captured by the ranking of j by i at time t, we constructed a measure of transitivity, τ, for the triple $\{ijk\}$ at time t where $\tau_{ijkt} = 1$ if $d_{ikt} \leq \max[d_{ijt}, d_{jkt}] + 1$, and 0 otherwise.[2] By summing over all triples $\{ijk\}$, $\tau = \sum \tau_{ijkt}$. Initially, transitivity was close to zero but it then climbed steadily. By week 3, the amount of transitivity was significantly greater than zero and it peaked in week 9 and remained there for the remaining weeks.

For an assessment of balance theory we coded the top four ranks to 1 and the bottom three ranks were recoded to -1. All of the remaining ranks were coded as 0. Using the Doreian and Mrvar (1996) partitioning method – based on the structure theorems of Cartwright and Harary (1956) and of Davis (1967) – we sought those partitions into plus-sets that minimized a criterion function $I_b = \sum_p + \sum_n$ where $\sum_p$ is the number of positive lines between plus-sets and $\sum_n$ is the number of negative lines within plus-sets. Consistent with the primary substantive structural balance hypothesis, the amount of imbalance, I_b, declined steadily.[3]

These results were pleasing as they revealed three coupled processes with *different time scales*. A general norm of reciprocity (Gouldner, 1960) makes "appropriate" behavior easy to learn and because the amount of reciprocity remains fixed, no further learning is required of the actors. Transitivity is a little more complicated and takes more time to learn. Although it is not clear whether there are such things

[1] We used permutation methods, see Krackhardt (1988).

[2] We needed to tackle a particular problem with these ranked data. If i chose j as the top ranked alter, and if j chose k as its first choice, transitivity is impossible for this triple. Because i has chosen j, this actor cannot choose k as the top ranked alter. But if i were to choose k as the second ranked alter, this would preserve transitivity under the definition of τ. This also meant that we did not have to recode the ranks for transitivity.

[3] If an unsmoothed measure is used, I_b drops steadily through the first 11 weeks after which, essentially, it is steady (but with the smallest value at week 14). If a smoothed measure is used, imbalance drops through the first 14 weeks.

as "transitivity norms" or whether there is simply a tendency towards transitivity in human groups, some learning is involved. In these data, learning to behave in ways that are more consistent with transitivity lasted about eight weeks. Thereafter, the amount of transitivity was fixed. Structural balance is even more complicated. Learning to behave in the context of the unclear partitioned structure(s) into plus-sets and adjusting ties in conformity to balance processes, as other people in the network are simultaneously adjusting their behavior, takes much more time to learn. Of course, an alternative interpretation is possible: people interact and respond to individual attributes in ways that generate reciprocity, transitivity and balance. See, for example, Feld and Elmore (1982) and Zeggelink (1993). Alas, as actor attributes have been lost for the Newcomb data, the two interpretations cannot be compared in these data.

Coupling these three processes is a much more difficult problem. We take some tentative steps in this direction by focusing on balance and signed transitivity.

2 SIGNED TRANSITIVITY

Transitivity is a vexing topic with two variants. One variant is based on the work of Davis and Leinhardt (1972) and their proposal of ranked-clusters as a general model of stratified sociometric systems. In their model, there are cliques whose members are all reciprocally linked. These cliques are distributed across a number of levels where all of the asymmetric ties go in the same direction between levels. In the case of affect ties, these ties go upwards from less popular actors to more popular actors. Holland and Leinhardt (1971) provide a formal treatment of these transitive graphs. A perfect ranked-clusters model implies that all of the triads in the network take particular forms. Transitive and vacuously transitive triads are permitted but all of the intransitive triads are not. See also Hallinan (1974) for an extended discussion.

Empirically, networks do not take this perfect form. As a result, these types of systems have been analyzed by counting triads. The triads are characterized by the number of mutual (M), asymmetric (A) and null (N) ties that they contain. Permitted triads (transitive and vacuously transitive) are expected to occur more frequently than chance while the intransitive triples are expected to occur less frequently compared to chance. Some of the difficulties inherent in these comparisons are revealed by the exchange between Hallinan (1982) and Feld and Elmore (1982).

We contend that, from a structural balance perspective, this variant of transitivity was developed by a sleight of hand trick with regard to balance. Davis and Leinhardt (1972, p. 225) in classifying triads into permitted and not permitted categories state that their "argument draws heavily upon the theorem of clusterability."[4] Their argument starts "by altering the notation of the lines (edges) so that M and N relations are 'positive' and the A relations are 'negative'." Signed ties are defined out of existence and Davis and Leinhardt proceed with only positive ties. The same is true for Hallinan (1974). As many network data sets are available with information about only positive ties, this approach has lead to many innovative methods for analyzing network data with only positive ties. While we have no objection to this line of analysis as a general approach to studying positive ties; our concern here is with *signed relations*. This brings us to *signed transitivity* as the second variant of transitivity.

Even though the measure of imbalance, I_b, used by Doreian *et al.* (1996) is a useful measure that facilitates an assessment of whether there is movement overall towards balance, it pays no attention to the configurations of triples. Yet Heider's (1946) formulation of balance theory is one that stresses "expected" outcomes for *triples* over time.[5] Rapoport (1963, p. 541) expresses some aphorisms based on Heider (1946) and Cartwright and Harary's (1956) generalization of Heider's formulation: (a) a friend of a friend is a friend (if $i \rightarrow j$ and $j \rightarrow k$ are positive then $i \rightarrow k$ will be positive), (b) a friend of an enemy is an enemy (if $i \rightarrow j$ is negative and $j \rightarrow k$ is positive, then $i \rightarrow k$ will be negative), (c) an enemy of a friend will be an enemy (if $i \rightarrow j$ is positive and $j \rightarrow k$ is negative, then $i \rightarrow k$ will be negative), and (d) an enemy of an enemy will be a friend (if $i \rightarrow j$ and $j \rightarrow k$ are negative, then $i \rightarrow k$ will be positive). Our concern is whether these plausible claims are supported in the Newcomb data as recoded by Doreian *et al.* (1996) to capture positive,

[4] This theorem is that of Davis (1967).

[5] Hummell and Sodeur (1990) make the useful distinction between triads and triples. Our attention here is focused on the triples $\{ijk\}$ and not the full triads.

TABLE 1
Pre-Transitive Conditions and Balance

Triple	Tie 1 $i \rightarrow j$	Tie 2 $j \rightarrow k$	Tie 3 $i \rightarrow k$	Balance
LLL	+	+	+	yes
LLD	+	+	−	no
LDL	+	−	+	no
LDD	+	−	−	yes
DLL	−	+	+	no
DLD	−	+	−	yes
DDL	−	−	+	yes
DDD	−	−	−	yes/no

null and negative ties. If there is a movement towards balance, it is reasonable to ask if there is movement so that balanced triples become more frequent and imbalanced triples become less frequent.

To pursue this, we construct a "pre-transitive" condition where $i \rightarrow j$ and $j \rightarrow k$ are specified as combinations of signed ties. The "post transitive" response is the $i \rightarrow k$ tie which can be positive or negative. The alternative outcomes are shown in Table 1. In the left column, there is a label for the types of triples where the first two letters are for the pre-transitive condition and the third is for the post-transitive condition. We use the label L (for "likes") as a generic label for a positive tie. Similarly, D (for "dislikes") is a generic label for negative ties. There are a total of eight combinations of $i \rightarrow j$, $j \rightarrow k$ and $i \rightarrow k$ ties. The column on the right of Table 1 indicates the balance/imbalance state of the triple. The ambiguity of DDD, the all negative tie triple, stems from the difference between the 2-balance of Heider where the all negative triple is imbalanced and k-balance[6] of Davis where $k \geq 2$. Johnsen (1986) provides an extended discussion of the logical implications of balance processes at the micro level and the way they generate macro level partitions (into plus-sets) when a network is balanced. Our concern here is much more empirical – consistent with Hummell and Sodeur (1990) – with a focus on triples[7] in signed networks that are not balanced.

[6] A signed network is k-balanced is the vertices can be partitioned into k clusters such that all positive ties are within clusters and all negative ties are between clusters.

[7] Hummell and Sodeur consider triads that can be expressed in terms of a set of eight triples. As indicated earlier, we restrict our attention to triples.

3 ANALYSIS

Within balance theory, the primary concern is with the hypothesized movement of the signed network towards balance. The basic hypothesis, cast in terms of triples, is very simple: through time, balanced triples will become more frequent while imbalanced triples will become less frequent. The traditional ways of pursuing this are flawed seriously. Our previous use of the line index of balance meant that we paid no attention to triples. Using proportions of positive triples to all triples comes a little closer to what we need. However, in that type of counting, all that matters is the *sign* of the triples. Here, we examine the *types* of triples and examine their frequency distributions through time.[8] The number of negative ties in a triple, by itself, is a poor measure if the intent is to examine movement towards balance.

Counting the incidence of types of triples is straightforward. Deciding if a particular frequency is "high" or "low" involves more effort. There are alternative null models for doing this. Here, we are constrained by the recoding of ranks to 1, -1 and 0 in a fashion where each actor has fixed out-degrees for both positive and negative ties. We "simulated" the construction of triples 10,000 times in the following fashion. For each simulation the rankings of each row of the sociomatrix were assigned at random. The top four ranks were assigned values of 1, the bottom three were assigned values of -1 and the remaining ranks were coded to zero. The counts of the triple types in Table 1 were done in the same fashion as for our transformed version of the Newcomb data. The empirical counts were then compared with the simulated counts.[9] This is shown in Figure 1.

The horizontal axis is time (counted in weeks) and the vertical axis is marked "p(obs > random)". It measures the proportion of times the observed count of a particular triple type was higher than in the corresponding simulated (random assignment of ranks) data. Working

[8] The entire set of triples are not independent. For a given i and k there are many $(n-2)$ potential j's for ijk triples. We count *all* triples as, empirically, all actors are involved in such triples. That different actors (as j's) are in the triples is not a problem as the operation of forces in all triples generate movements towards balance – *if* there are balance theoretic mechanisms at work. Nor is it a problem to use different permutations of i, j and k in triples.

[9] Alternative null models could have been specified in ways that incorporate the operation of other mechanisms. One such null model could have taken into account differential popularity and differential unpopularity.

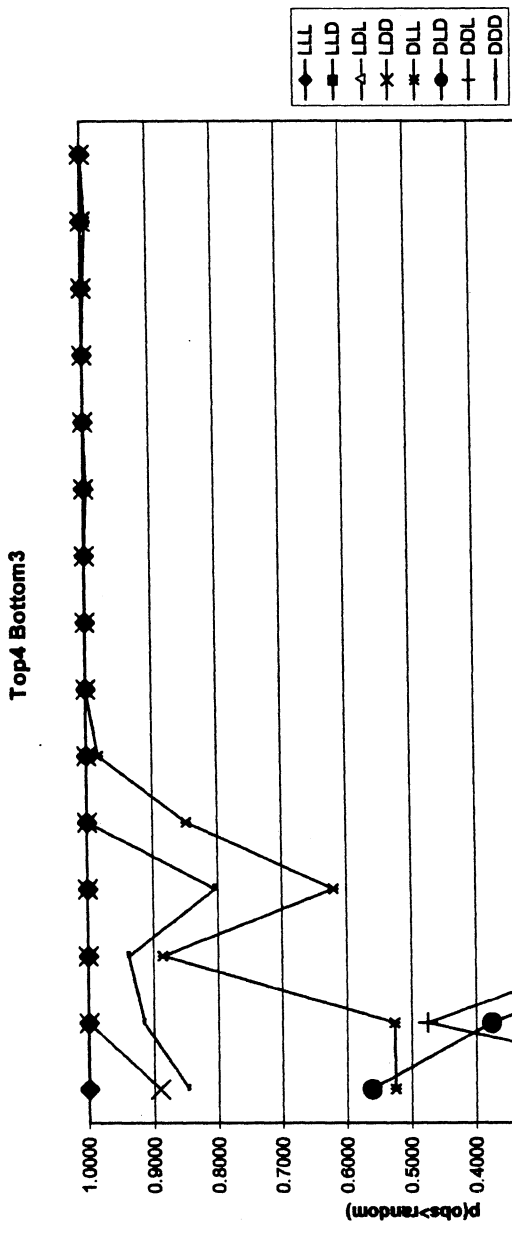

FIGURE 1 Probabilities that the number of observed types of triples are greater than random.

with a significance level of $\alpha = .05$, if this count goes above 9,500 or below 500, the number of triples counted is significant. If the proportion is 0, the empirical counts are *never* higher than the counts in the simulated random world data. Similarly if the proportion is 1, the empirical counts are *always* higher than the simulated random world's data. The trajectories in Figure 1 show the movement on these proportions through time.

These trajectories are labeled: $(LLL, \blacklozenge)$; $(LLD, \blacksquare)$; $(LDL, \triangle)$; $(LDD, \times)$; $(DLL, *)$; $(DLD, \bullet)$; $(DDL, +)$ and $(DDD, -)$. The most noticeable feature of Figure 1 is that *all of the trajectories reach the extreme values of 1 or 0 well before* the end of the study period. The *LLL* triple proportion starts at 1.0 and never departs from that value. The *LDD* triple is slightly below 0.9 at week 1 but reaches 1.0 at week 2 and remains there. Both of these trajectories involve balanced triples and support strongly the structural balance hypothesis. The trajectory for *LDL* starts at 0.0 and remains there. The initial value of the proportion for *LLD* is very close to zero (and is "significant") at the outset and reaches zero at week 2. Both of these trajectories are for imbalanced triples: again, the structural balance hypothesis is supported strongly.

The straightforward structural balance hypothesis does not fare well for three of the other four triple types. Consider *DLL*. It starts at a probability close to 0.5 (not significant) and (with a spike at week three) moves *towards* the extreme value of 1.0. From week 6 onward its frequency is always greater than would be expected by chance. But this triple is *imbalanced.* Clearly, the temporal version of the structural balance hypothesis is not supported for this type of triple in these data. The same holds for the *DDD* triple although its trajectory starts at a much higher value. It, too, reaches the extreme of 1.0 but does so (in week 5) before the *DLL* triple. However, we note again, that in the Davis (1967) framework, the *DDD* triple is defined as balanced. More consequential for the structural balance thesis are the trajectories for *DLD* and *DDL*. Both are balanced triples and yet their proportions move *towards* zero. They are *less* frequent than would be expected under random conditions and become significant in week 5.[10] Both of

[10] While the *DDL* triple starts at a value that is slightly above 0 – and is significant – it departs dramatically from 0 in week 2. The two tiny departures of the *DLD* trajectory at week 6 and week 11 seem irrelevant to the general result.

these trajectories are *for balanced* triples and they become *rare* during the passage of time.

Figure 1 suggests that the essential dynamics are over by weeks 5 or 6. However, all that can be discerned from Figure 1 is that specific triple types are *plentiful or rare relative to chance*. If we focus on the actual counts, there are patterns beyond the crossing of certain thresholds. Let d represent the number of times there is a specific pre-transitive condition where $i \rightarrow k$ makes the triple balanced and let b be the number of times where the same pre-transitive condition has $i \rightarrow k$ take the form inconsistent with balance. The conditional probability of the $i \rightarrow k$ completing the triple in a way that is balanced, given the existence of the pre-transitive condition, is $(d/(b+d))$. We denote this by p_{bal} for our subsequent discussion. This is computed for each of the triple types in Table 1.

The plots of these trajectories through the 15 time points are shown in Figure 2. The nature of the trajectories will be described shortly. There is an additional complication to consider before doing so. Let N_L, N_D and N_A respectively denote the number of positive (L) ties from an actor, the number of negative (D) ties from an actor and the total number of number of other actors in the network. (In this case $N_L = 4$, $N_D = 3$ and $N_A = 16$ for all actors.) Now consider the $i \rightarrow j$ tie. Given the existence of this tie, it is possible to determine the probabilities of the $i \rightarrow k$ tie for random choices conditioned on the presence of an $i \rightarrow j$ tie. These are expected values, EV, for probabilities and are provided in Table 2. The trajectories shown in Figure 2 display the "adjusted" trajectories where $\delta = (d/(b+d)) - EV$. In a simpler labeling, this is $\delta = (p_{\text{bal}} - EV)$.

Our attention will be confined to an assessment of the "fundamental structural balance hypothesis" which we contract to "FSBH". This has two versions: (i) balanced triples will be present and (ii) the number of balanced triples will increase through time and the number of imbalanced triples will decrease through time. While the former is

TABLE 2
Probabilities of the $i \rightarrow k$ given $i \rightarrow j$ Under Random Choices

	L	D
L	$EV = (N_L - 1)/(N_A - 1) = 3/15$	$EV = (N_D)/(N_A - 1) = 3/15$
D	$EV = (N_L)/(N_A - 1) = 4/15$	$EV = (N_D - 1)/(N_A - 1) = 2/15$

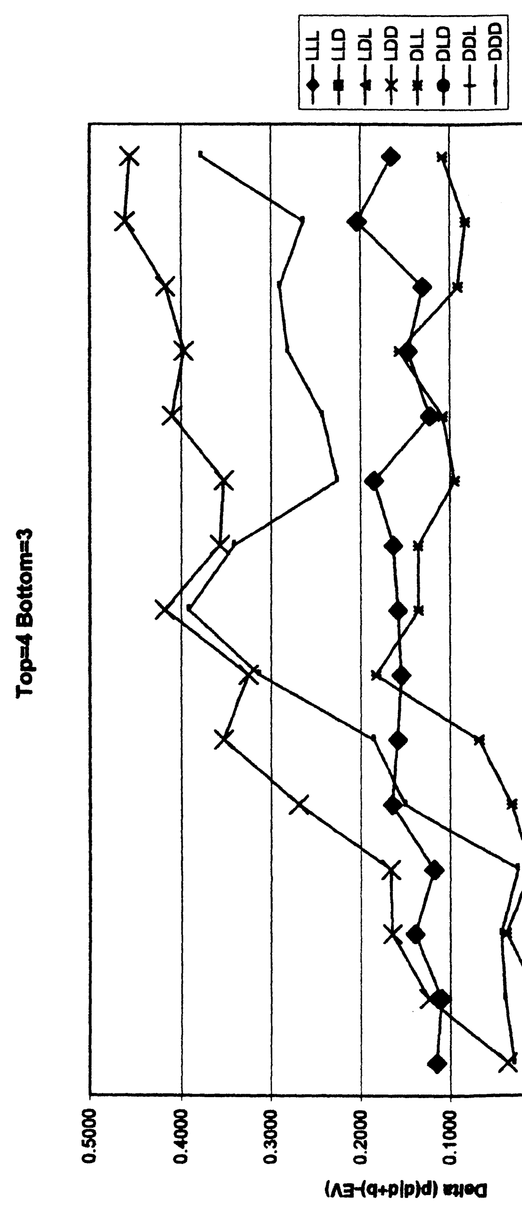

FIGURE 2 Trajectories giving (probabilities – expected probabilities) for eight triple types.

trivial, the latter is consequential. We consider first the balanced triples.

For the *LLL* triple in Figure 2 starts with δ at a value that is just above 0.1. This changes little through the study period ending up at a value slightly below 0.2. (The corresponding values for p_{bal} are slightly above 0.3 to close to 0.4.) The FSBH is confirmed but the *small* increase in δ (and p_{bal}) could be viewed as surprising. The trajectory for the *LDD* triple is unequivocal: it starts at a modest value just above 0.0 (p_{bal} above 0.2) and climbs steadily to a value close to 0.5 (p_{bal} close to 0.7) in Figure 2. The FSBH is confirmed convincingly. This is not the case for two other types of balanced triples. For *DDL*, δ starts above -0.1 and generally *diminishes* in value throughout the study period. (The value of p_{bal} starts at 0.2 and declines to 0.1.) This does *not* support the FSBH in its second form. For the *DLD* triple, the value of δ (and p_{bal}) *also diminishes* throughout the study period and reaches a value below -0.1 (slightly above 0 for p_{bal}). For balanced triples, the evidence concerning the FSBH is mixed: the incidence of two types of balanced triples (*LLL* and *LDD*) increases while for the other two triple types (*DDL* and *DLD*) the incidence decreases. And, as noted above, the change for the *LLL* triple type is modest.

We now consider the imbalanced triples. The value of δ for the *LLD* triple starts at a point around -0.05 and diminishes to a value close to -0.2 by week 15. (The corresponding values for p_{bal} are 0.15 and 0.) Similarly, for the *LDL* triple, δ starts with a value slightly above -0.1 (p_{bal} above 0.1) and diminishes steadily to a value even closer to -0.2 (p_{bal} is close to 0). The two trajectories remain close to each other. For both, the FSBH is confirmed: the incidence of these imbalanced triples diminishes through time. For the two other imbalanced triples (as defined by Heider (1946)), the FSBH is contradicted. For the *DLL* triple type in Figure 2, δ starts at a value close to 0 (p_{bal} slightly below 0.3) and increases modestly to a value just above 0.1 (p_{bal} is slightly under 0.4). For the *DDD* triple, the evidence is even more striking: it starts with δ just above 0 (p_{bal} just above 0.3) to a value below 0.4 (p_{bal} below 0.7) in Figure 2. As is the case for the balanced triples, the incidence of two of the imbalanced triple types (*LLD* and *LDL*) have trajectories supporting the FSBH while for the other two types (*DLL* and *DDD*) the evidence contradicts the FSBH.

If we move from balance as defined by Heider to balance as defined by Davis (1967) the evidence from the *DDD* data supports strongly the modified FSBH when *DDD* is defined as balanced. In general, what can we say about the evidence for the FSBH? First, the evidence *differs according to the type of triple considered.* Second, our fine grained consideration of triple types suggests that the overall movement towards balance is a *net change* over the changes in differing triple types. Overall, the change in the incidence in triples consistent with the FSBH is larger than the change in the triples inconsistent with the FSBH – at least in these data. The shifts in the trajectories of δ (and p_b) are greater and, overall, the count of balanced triples consistent with the FSBH increases and exceeds the counts of triples inconsistent with it. The latter types of triples decrease in time. (For k-balance with $k > 2$, support for the FSBH is much stronger as the incidence of the *DDD* triple increases dramatically through time.) Third, "movement towards balance" results from different off-setting mechanisms that are lumped together when only the amount of balance – however it is computed – is considered. The implications are twofold: (1) the balance theory ideas are flawed and (2) there are other mechanisms operating.

4 A CLOSER LOOK AT STRUCTURAL BALANCE

The results of our simple minded count of the through time frequencies of the eight triple types described in Table 1 are easy to describe. We do this in Table 3. First, we consider the balanced triples. The *LLL* triple count is always present (in numbers higher than expected by chance) and supports the weak form of the FSBH. However, it is not a triple type that increases through time – hence the use of "supports" in parentheses in Table 3. The trajectory for the *LDD* triple in Figure 3 shows the greatest movement, a result that supports the FSBH. It seems that a large part of the movement towards balance comes from the increase in the frequency of this triple type. The presence of both the *DLD* and the *DDL* triples diminishes through time and contradicts the FSBH.

For the imbalanced triples, the frequencies of both the *LLD* and *LDL* triples decrease through time and the evidence from these triples supports the FSBH. The number of *DLL* triples should diminish through time according to the FSBH. Empirically, the reverse occurs. For 2-balance, the increase in the frequency of *DDD* triples does not support the FSBH. However, if *DDD* is defined as balanced the k-balance (with $k > 2$) version of the FSBH is supported. Our task, now, is to make sense of these results.

Thus far, our discussion has been in terms of balance theoretic ideas. Indeed, we have given primacy to balance theoretic ideas. As we had expected unequivocal support for the FSBH, it is clear that our initial formulation is incomplete. As noted above, all data concerning individual attributes (and spatial features like shared rooms and rooms on the same floor) have been lost for the Newcomb data. Although we cannot use absent information, it seems that actor attributes have to be relevant. The arguments of Zeggelink (1993), Stokman and Zeggelink (1996), van de Bunt (1999) and Newcomb (1961) imply that actor attributes have far more importance than the forgoing discussion has allowed. We suggest three ways of interpreting our results, two of which hinge on (potential) actor attributes: (i) by focusing on i and considering a competition mechanism, (ii) focusing on k and considering the attributes of k and some group consensus concerning a small number of specific k's, and (iii) examining the movement of the macro-structure of the group through time.

4.1 Examining the Ties of i

One interpretation of our results starts with the nature of the $i \rightarrow j$ tie. All that seems to matter is the sign of that "first" tie:

TABLE 3
Pre-Transitive Conditions, Balance and Evidence

Triple	Tie 1 $i \rightarrow j$	Tie 2 $j \rightarrow k$	Tie 3 $i \rightarrow k$	Balance	Supports Balance
LLL	+	+	+	yes	(supports)
LLD	+	+	−	no	supports
LDL	+	−	+	no	supports
LDD	+	−	−	yes	supports
DLL	−	+	+	no	contradicts
DLD	−	+	−	yes	contradicts
DDL	−	−	+	yes	contradicts
DDD	−	−	−	yes/no	(supports)

1. *For all of the triple types where the first $(i \rightarrow j)$ tie is positive, the fundamental structural balance hypothesis is supported;*
2. *If the LLL triple is defined as balanced, there is additional strong support for the (Davis version of the) fundamental balance theory hypothesis; and*
3. *For the other triple types with $(i \rightarrow j)$ tie negative, the fundamental structural balance hypothesis is contradicted.*

To pursue the implication of these results, we examine the four triple types where the FSBH was not supported by considering pairs of outcomes. Consider the *DLL* and the *DLD* triples. They both have $i \rightarrow j$ negative and $j \rightarrow k$ positive. Structural balance predicts the post-transitive condition of $i \rightarrow k$ as negative. But instead of fewer *DLL* triples and more *DLD* triples we get exactly the reverse. An alternative mechanism can take the following form that assumes that i dislikes j and knows that j likes k. If i is also inclined to like k, then i and j become rivals for the attention of k. We suggest that the "balance mechanism" of i disliking k to reach balance is dominated by a "competition mechanism" where i and a disliked rival j compete for k. This would account for the increase of imbalanced triples (of the *DLL* type) and a decrease of balanced triples (of the *DLD* type). Put differently, while there are balance mechanisms, they are not the only mechanisms and need not be the dominant mechanisms. This has clear implications for the collection of data as the idea of "being rivals for k" requires data to permit that kind of interpretation.

Table 4 shows the partition structures reported by Doreian *et al.* (1996) for weeks 1 and 15. The *DDL* and *DDD* pair of mechanisms can be examined in two ways. They each have the pre-transitive condition of $i \rightarrow j$ and $j \rightarrow k$ both being negative. Balance theory (as defined by Heider) predicts that the triple will be completed by a positive $i \rightarrow k$. There would be increases through time of the number of *DDL* triples and decreases in the number of the *DDD* triples. We get the reverse. Already we have alluded to the operation of a Davis balance theory mechanism. If the macro-structure of the group has at least three plus-sets of actors with mutual hostilities, there has to be a presence of higher numbers of *DDD* triples. The network displayed in Table 4 (lower panel) has exactly this structure. In our earlier work we reported the Rapoport (1963) aphorisms. For these data, the ideas that "a friend of

TABLE 4
First and Final Partition Structures for the Newcomb Data

Partition into Three Clusters at First Time Point

	A	F	H	M	B	D	G	N	P	C	E	I	J	K	L	O	Q
A	0	1	−1	1	0	0	0	0	0	0	0	−1	−1	1	0	0	1
F	0	0	1	1	0	1	0	0	0	0	−1	1	−1	−1	0	0	0
H	0	1	0	1	0	0	−1	−1	0	−1	0	0	1	1	0	0	0
M	1	1	−1	0	−1	0	0	0	0	−1	1	0	0	0	0	1	0
B	0	0	−1	0	0	1	1	0	1	−1	0	0	0	−1	0	0	1
D	0	1	−1	0	1	0	1	0	0	−1	−1	0	0	0	0	0	1
G	−1	0	0	−1	1	1	0	0	0	0	−1	0	0	0	1	0	1
N	−1	0	−1	−1	0	0	1	0	0	0	0	1	1	0	0	1	0
P	0	−1	0	0	0	1	−1	0	0	−1	0	1	0	1	0	0	1
C	0	0	−1	−1	0	0	0	0	−1	0	0	0	0	1	1	1	1
E	1	1	1	0	0	0	0	−1	0	0	0	0	0	1	1	0	1
I	0	0	−1	0	−1	−1	1	0	0	0	0	0	0	1	1	0	1
J	1	1	0	0	−1	−1	1	0	0	0	0	0	0	−1	0	1	0
K	0	−1	−1	0	0	0	0	−1	0	1	0	1	0	0	1	0	1
L	−1	−1	0	−1	0	0	0	0	0	1	0	0	1	1	0	0	1
O	−1	0	0	0	0	0	0	−1	−1	1	1	0	1	1	0	0	0
Q	0	0	0	0	−1	1	0	−1	0	0	1	1	0	0	1	−1	0

Four Cluster Partition at Final Time Point

	A	B	D	E	F	G	H	I	K	L	M	N	Q	C	J	O	P
A	0	0	0	0	0	0	1	0	0	0	1	1	1	−1	−1	0	−1
B	0	0	1	1	0	0	0	0	0	1	0	0	1	0	−1	−1	−1
D	0	1	0	1	1	0	0	0	0	0	0	−1	1	−1	−1	0	0
E	0	1	1	0	0	0	0	1	0	0	1	0	0	0	−1	−1	−1
F	0	0	1	0	0	0	1	1	0	0	1	0	0	−1	−1	0	−1
G	0	1	0	−1	0	0	0	0	1	1	0	0	1	0	−1	0	−1
H	1	0	1	0	1	0	0	0	0	0	1	0	0	−1	−1	0	−1
I	0	0	1	0	1	0	0	0	0	0	0	1	1	0	−1	−1	−1
K	0	1	1	0	0	0	−1	0	0	1	0	0	1	0	−1	0	−1
L	0	1	0	0	−1	1	0	1	0	0	0	0	1	0	−1	−1	0
M	1	0	0	0	0	1	1	0	0	0	0	0	1	−1	−1	0	−1
N	1	0	0	−1	1	0	0	1	1	0	0	0	0	−1	−1	0	0
Q	1	1	1	0	0	0	0	1	0	0	0	0	0	−1	−1	0	−1
C	0	0	0	0	1	0	0	1	0	1	0	−1	1	0	−1	0	−1
J	1	0	0	0	0	1	1	0	0	1	−1	0	0	−1	0	0	−1
O	0	0	1	−1	0	0	0	1	−1	0	0	0	1	0	−1	0	1
P	0	0	1	0	0	0	−1	1	0	1	0	0	1	−1	−1	0	0

a friend is a friend" and "an enemy of a friend is an enemy" receive support. However, it seems that *neither* "a friend of an enemy is an enemy" *nor* "an enemy of an enemy is a friend" are supported in these data.

4.2 Focusing on k[11]

The argument of the previous section tries (perhaps desperately) to preserve a structural argument by keeping a focus on the triples and appealing to balance theoretic ideas. A *much simpler* argument can be based on the pair of $j \rightarrow k$ and $i \rightarrow k$ ties. Instead of thinking in terms of balance theory, we focus on the triple types for the upper four trajectories in Figure 2 as they involve the larger values of δ (and p_{bal}). They are the *LLL*, *LDD*, *DLL* and *DDD* triples. The striking communality of these four trajectories is that they each finish with the same pair of tie types. *The triples with the higher and/or increasing values of* p_{bal} *are those where i and j agree in their affective ties with k.* They either both like k or they both dislike k – regardless of whether i likes j. And if we switch the location of i and j in the triple, they agree regardless of whether j likes i. Thus, i and j can agree in their views of k regardless of their views of each other. This suggests that balance is either irrelevant or is dominated by a stronger mechanism. *Actors such as k have attributes that are recognized and assessed in the same way by other social actors in the group.* This is clearly a *non-structural* account of the generation of the $i \rightarrow k$ tie. This provides an echo of the Feld and Elmore (1982) results in which they suggest that inequality in popularity (for networks with only positive ties) can generate transitive triples that are mistaken for the operation of a transitivity mechanism. Our results can be viewed as extending this to signed networks. This line of thought is consistent with that of Zeggelink (1993) and van de Bunt (1999) who argue that there are subjective advantages to forging ties with alters "who think alike" and that both balance and transitivity mechanisms are secondary to more direct mechanisms by which friendship ties are formed.

[11]We use k as a generic label for the third vertex in an (ijk) triple. In the reported results, the vertex K is a specific label for one of the actors.

4.3 Macro Structure and Another Look at *k*

Table 5 shows the distribution of negative and positive ties as they are *received* by each of the actors. Each row corresponds to an actor with each column representing a time point. The figures for the first and last time points can be confirmed from the information provided in Table 4. Some features of Table 5 are noteworthy. At the first time point, the distribution of received negative ties ranges from 0 to 8. Only two actors, *L* and *Q*, receive no negative ties and, through time, they never receive negative ties. Actor *D* starts by receiving only two negative ties and from the second week on never receives more that one negative tie and receives none for nine time points. Actor *I* seldom received negative ties and after week 9, *I* joined *L* and *Q* in receiving no negative ties. These all are actors who are not disliked and they also receive more positive ties than the other actors. See Table 5 (lower panel).

At the other extreme, *J* ends up as disliked by everyone – as is evident in Table 5 (top panel). Although he started by receiving only two negative ties, there were 8 actors disliking him at week 2. By week 4, he was disliked by 11 others and was identified as a truly disliked singleton in the balance partition. Actor *P* had a less extreme fate. Like *J*, he started by receiving only two negative ties. At week 2, he received 6 negative ties and, by the end of the study, received 12 negative ties. Actor *C*, was disliked by at least 4 other actors and, by week 7, received 9 negative ties. This remained essentially unchanged for the rest of Newcomb's study. These three actors were recipients of 37 negative ties (out of a total of the 51 negative ties) at the end. They are joined by *O* in Table 5 (top panel) as another singled out as a recipient of negative ties. This is more true for weeks 11 and 12 when he received 9 negative ties. We note that in week 12, actors *C*, *J*, *O* and *P* received an astounding 43 negative choices (out of the 51 such choices). None of these actors receive more than one positive tie in week 12.

The set of actors who seldom or never receive negative choices (and receive more positive choices) and the set who received many negative choices (and very few positive choices) are consistent with the argument in the previous subsection: the other actors seem agreed in their affective choices regarding the very popular and the very unpopular actors. As examples of *k*, the actors in the set

$\{D, L, Q\}$ are viewed consistently as are the actors in the set $\{C, J, O, P\}$. The non-structural account is that they have attributes or behaviors that the majority of actors can recognize regardless of the signed triples within which they are located. However, the processes or mechanisms may be more subtle in their operation. Initially, actor H received *more* negative ties at week 1 than anyone else and was clearly disliked. While this persisted through week 5, the number of negative received ties by H dropped dramatically thereafter.[12] Actors A and N both start by receiving 5 negative ties, the second most extreme volume after that received by H. The received number of negative ties then dropped for both actors. If we advance the argument that $\{C, J, O, P\}$ have attributes that make them disliked consistently, we cannot make that claim for actors in the set $\{A, H, N\}$ who started among the actors receiving more negative choices than anyone else. In short, we doubt that the receipt of negative ties is driven *solely* by individual attributes. Consider the folk notion that "people are judged by the company they keep". To the extent that this has merit, individual attributes are not the sole basis on which positive and negative ties are determined.

If we look at the figures shown in Table 5, there is evidence of a polarization process that leads to a stark distribution of the receipt negative ties: some actors that have large counts while for the majority of actors, there is movement towards receiving none or few negative ties. This almost bimodal distribution may well be driven, in part, by balance mechanisms. Being disliked is *not* just a function of actor attributes. It can be viewed in terms of the subgroups – i.e. plus-sets – to which individuals belong. The partition for week 15 shown in Table 4 has one large plus-set with many positive ties within the plus-set and virtually all of the negative choices of the members of the plus-set going to the massively disliked set of actors in $\{C, J, O, P\}$. That these actors also tend towards disliking each other is consistent with the build up of the number of *DDD* triads through time that was described earlier. *In part*, then, dislike is contextually learned through membership in plus-sets and the operation of balance.

[12] The number of positive ties received by H remain modest through time.

TABLE 5
Receipt of Positive and Negative Ties Through Time

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Negative Ties															
A	5	4	4	3	2	1	1	0	0	1	1	0	0	0	0
B	4	4	1	3	4	1	0	0	1	0	1	0	1	0	0
C	5	5	4	5	5	6	9	10	10	9	9	9	9	8	9
D	2	1	1	1	0	1	0	0	0	1	0	0	0	0	0
E	3	1	0	2	1	1	1	0	1	1	1	0	0	2	3
F	4	4	4	4	4	3	1	3	3	3	1	1	2	1	1
G	2	4	5	4	2	0	0	0	1	0	1	1	2	1	0
H	8	7	8	8	6	2	1	1	0	3	2	4	0	2	2
I	1	1	2	0	1	1	1	1	2	0	0	0	0	0	0
J	2	8	8	11	12	13	13	15	14	11	13	11	13	14	16
K	3	1	0	2	0	2	1	1	1	1	0	2	2	2	1
L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M	4	0	1	1	2	2	3	2	0	0	1	1	1	1	1
N	5	4	4	1	0	0	2	1	2	2	0	1	1	1	2
O	1	1	2	1	3	6	6	5	5	7	9	9	7	6	4
P	2	6	7	5	10	12	12	12	11	12	12	14	13	13	12
Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Positive Ties															
A	2	4	4	5	5	3	7	5	6	6	6	5	5	7	5
B	2	2	4	4	4	4	4	4	4	4	3	3	5	5	6
C	3	4	2	2	1	1	0	1	0	1	2	0	1	0	0
D	5	7	7	8	6	7	7	7	7	5	7	7	6	6	9
E	3	5	2	2	4	5	2	2	1	2	3	2	1	1	2
F	5	3	6	6	6	7	7	6	9	7	6	7	7	6	5
G	5	6	5	5	3	6	4	5	2	4	4	2	3	5	3
H	2	0	2	2	3	3	2	3	3	4	2	2	3	4	4
I	5	8	7	8	8	8	10	11	10	9	9	10	10	9	8
J	4	3	1	2	0	0	0	0	0	0	0	0	0	0	0
K	8	3	3	1	1	1	0	1	2	1	3	2	2	0	2
L	6	4	4	4	8	5	7	5	7	7	5	6	5	6	6
M	3	5	5	5	4	4	3	3	3	4	3	5	5	5	4
N	0	2	4	3	3	1	3	2	3	2	3	3	3	2	2
O	4	2	2	1	1	1	0	1	0	0	1	1	1	1	0
P	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1
Q	10	9	10	9	10	11	11	11	10	11	10	12	10	11	11

5 CONCLUSION AND DISCUSSION

We have distinguished the eight types of triads defined by Heider (1946) in his formulation of balance theory. This was done to provide a fine-grained examination of the "Fundamental Structural Balance Hypothesis" (FSBH) that human signed networks tend towards balance through time. We found that the counts of the *LLL*, *LLD*, *LDL* and *LDD* triads changed in the direction that was consistent with the FSBH. However, we found that the counts of the *DLL*, *DLD*, and *DDL* did not move in a direction that is consistent with the FSBH. The counts for the *DDD* triads were ambiguous. For the strict 2-balance definition of Heider, this movement was inconsistent with the FSBH and for k-balance (where $k > 2$) the movement was consistent with balance as re-defined by Davis (1967).

A second plausible mechanism is that some actors, k, have attributes that are recognized and coded in the same way by pairs of other actors, i and j, regardless of whether i or j like each other. This mechanism has nothing to do with structural balance. The counts of some of the triples that "support" balance theoretic ideas could have been driven by differential popularity and differential unpopularity. If actor k is popular then finding triples (both *LLL* and *DLL* with $j \rightarrow k$ and $i \rightarrow k$ positive) to be more frequent is to be expected. Similarly, if k is very unpopular then finding the *LDD* and *DDD* triples to be more frequent could be driven by this unpopularity rather than structural balance.

Again, we have to stress the severe constraints dictated by the nature of the data and our recoding of the ranks. Some of the recoded ties – especially towards the end of Newcomb's study period – seem problematic. If every other actor dislikes J it seems unlikely that J will like four of these other actors. Even though J did rank the other 16 members of the pseudo-fraternity, it could be that he *disliked all* of the others or *did not like any* of them. If this is the case, regardless of whether he disliked the others or did not like any of them, the recoding of the top four ranks to positive values is a major data problem. Strictly, the rankings (of the remaining 16 actors by each actor) cannot be compared directly. It would be far better if we had explicit evidence for positive and negative ties rather than use a recoding of ranks.[13]

The third idea advanced here as a mechanism is that *some* of the macrostructure of a small group is generated by balance and that this macrostructure constrains the distribution of *some* of the ties in ways that are *consistent* with balance. When counting the incidence of types of triples and deciding if these counts are high or low, alternative null models could have been considered. One such model could have been

[13] We note that even though the Newcomb data are far from ideal, there are still signals that can be detected. And this gives us some faith in the findings we report.

conditioned by the receipt of both positive and negative ties. The problem with this is that primacy would then be given to differential popularity and differential unpopularity. It seems that through time generation of ties – and resulting group structures – comes through the operation of multiple mechanisms. Unfortunately, disentangling them in a definitive fashion lies well outside the scope of this study given the nature of the Newcomb data.

Future attempts to assess the FSBH must differentiate the triple types considered here. If this is not done, the meaning of the measures that purport to capture the "amount of imbalance" (or "balance") remain problematic.

If these measures capture anything, it is a broad description of "something" changing through time in a specified direction. Our argument here is that the "tendency towards balance" is the result of the operation of multiple mechanisms. Moreover, these mechanisms need not operate in ways that are consistent with each other. One such mechanism could involve "competition" in the *DLL* triad. Our argument is speculative but we cannot ignore the idea that some actors have attributes that generate "liking" and "disliking" ties that have little to do with structural balance. We do not know how to formally specify the coupling of a structural balance mechanism with a joint recognition of attributes mechanism. Clearly, structural balance cannot be all of the story and our results suggest it might not be the dominant story. We note that over time certain actors become more and more unpopular. Whatever mechanisms are operating, they generate "inequality" in the receipt of negative ties. This seems to parallel the Feld and Elmore (1982) observation concerning popularity. Further, it seems that the inequality in the receipt of negative ties is far more acute than the inequalities associated with the receipt of positive ties.

We do not deny that actor attributes are relevant for the formation of positive and negative ties. Without doubt, actor oriented models of the sort proposed by Snijders (1997) will be useful for studying these phenomena. But not with the Newcomb data as information about individual attributes have been lost.

That the sign of the first tie in the pre-transitive may be important is interesting. The simulations of Hummon and Doreian (2000) for actors, each with their own cognitive images, where a balance process operates in the sense of Heider, as well as a group structure with its own balance structure and process. Some of their evolved structures take a form where the group level structure is imbalanced while every actor's cognitive image of the structure is balanced. Such a termination point provides a simple argument for the creation of a group structure that need not be balanced even though balance theoretic processes are operating. The results reported here provide complementary evidence in the sense that, in the mind of i, the sign of that first tie to j is very important for the generation of the $i \to k$ tie. Unless, of course, the attributes of the third actor, k, have greater force in generating ties. At a minimum, structural balance is a more complicated process than most discussions allow. Our results suggest it is unlikely to be the only process that is operating. They may even suggest that it is not the most important process. Regardless, specifying how the mechanisms discussed herein are coupled to each other and how they operate in relation to each other becomes the next major task. This will require the collection of much better and more extensive data than is done usually by empirically minded structural balance theorists. An additional implication is that data collection by scholars not in the balance theoretic tradition is lacking also if negative ties are relevant. Thus, data collection in the future for *signed relations* requires that information concerning negative ties be obtained in addition to information about positive ties. Without data on both positive and negative ties – as well as information about individual attributes – disentangling the kinds of mechanisms considered here is impossible.

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