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Event sequences as generators of social network evolution

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Abstract

The argument presented in this paper is that one fruitful approach to the study of social network evolution takes the form of examining event sequences as generating mechanisms. Evidence for this comes from two empirical studies of structural balance theory and one simulation study of balance theoretic processes. Four views of causality—system, statistical (predictive), mechanism and algorithmic—are briefly contrasted and then examined with structural balance theory in mind. The conventional statement of the theory turns out to be under specified and inattentive to alternative mechanisms that can generate signed networks through time. Empirical studies of structural balance are limited with regard to the kinds of data that are usually collected. Proposals for studying the generation of signed networks through event sequences while being attentive to structural balance ideas are presented. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The term '*evolution* of social networks' suggests that (some) social networks change through the operation of coherent social processes. In turn, this suggests that substance is an important source when examining network evolution. One coherent substantive theory is structural balance theory. More importantly, it is a theory concerned with through time social network change for signed social relations. As such, it can provide a useful point of departure for examining social network evolution. This paper attempts to reconcile two very divergent assessments of this theory as a step towards understanding the evolution of signed structures. This reconciliation leads naturally to an examination of event sequences as potential generators of evolutionary change of network structures.

Davis (1979, p. 52) sees structural balance theory as 'nifty' in the sense of being "falsifiable, nonobvious and simple" (and therefore successful). In contrast, Opp's (1984) bleak

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assessment is that the theory failed and fell from favor because of insufficient empirical support. At best, the empirical evidence was mixed and inconclusive. Reconciling these seemingly inconsistent views rests on a series of ideas taken from social network analysis, a mathematical statement of structural balance theory, meanings of the term causality in a social network context, some recent empirical assessments of balance theory using longitudinal data and a computational simulation model of balance theoretic processes. These arguments are coupled to forge an argument that network evolution can be studied usefully in terms of *event sequences*.

2. Evolution of social networks

The most straightforward definition of a social network is in terms of a set of social actors, $\mathcal{V}$, and a social relation R. In formal terms, a network, $\boldsymbol{G}$, is defined as $\boldsymbol{G} = (\mathcal{V}, \mathcal{A})$ where $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$ and $R = \{\mathcal{A}_i \in \mathcal{A}\}$ is the social relation defined over the elements of $\mathcal{V}$. While the idea of a network can be extended straightforwardly to multiple relations, $\{R_j\}$, this discussion will be framed in terms of a single relation. Given the focus on structural balance, the 'like–dislike' relation is the primary example used here. Discussions of networks evolving can be cast in terms of relational ties changing through time. Doreian et al. (1997, p. 3) argue that "network processes are *series of events* that create, sustain and dissolve social structures" (emphasis added). For my purposes here, the crucial idea is the *sequencing* of events. While this leaves open the question of the nature of the mechanisms generating the sequences of network related events, an idea to be explored here is whether the sequences themselves form generating mechanisms. Stokman and Doreian (1997, p. 235), in an effort to outline some principles for analyzing social network evolution, made three observations that have relevance for this discussion. One is that for many network studies "the underlying process for network change is assumed to be located in the network structure". Their use of the term 'process' is, perhaps, unfortunate because it suggests that only one process operates. It is necessary to modify this formulation to include the possible operation of multiple processes.

In one of the empirical papers that is important for the empirical part of this discussion, Doreian and Krackhardt's (2001) empirical assessment of structural balance theory (see Sections 3 and 6) concluded that the balance theoretic process is better viewed as a set of multiple balance processes. Implicit in the notion of the structure of the social network at one time point conditioning the structure of the network at a subsequent time point is the idea of the network being a self-organizing system. It changes through time as a result of the operation of generating rules where the attributes of the actors are thought to play little or no part in the process.

A second observation from Stokman and Doreian (1997, p. 237) (for other network studies) is "the underlying process for network change is assumed to be located in characteristics of network members". Here, actor attributes matter and this qualifies the idea of an autonomous systemic self-organizing social network into one that is self-organizing while actors pursue network based goals. Consistent with the *intent* of Stokman and Doreian, considering *both* network structures and actor attributes in empirical analyses seems essential. While this is both a modest and unoriginal idea, Doreian (2001a) points out that there are major empirical problems in doing this. The simulation study informing the discussion in Section 7 provides an example of a multiple-level process.

Stokman and Doreian's (1997, p. 237) third observation, as an extension of the second, is that "some of these individual characteristics evolve over time as well". There is a sense in which the social network, as a structure, co-evolves with the actors as actor attributes both affect the structure and are affected by the structure. This seems particularly relevant in the context of structural balance theory. Another finding of Doreian and Krackhardt (2001) is that the actor attributes of being 'likable' or 'dislikable' also evolve through time under the operation of balance theoretic processes that also structure the pattern of signed ties in a social group. There is a sense also that the 'global' structure of the network is created as an emergent phenomenon through time in ways that are not reducible to terms of unchanging actor attributes and the actions of actors.

Stokman and Doreian (1997) also articulated six principles for constructing models that capture network evolution:

- P_1: Networks have an instrumental character for network members as these members have structured goals and some goals are achieved through network choices.
- P_2: Actors, at least in part, act on local information.
- P_3: There is parallelism whereby actors can act separately (but not completely independently).
- P_4: Models should be kept simple (at least initially).
- P_5: Models should have sufficient empirical referents.
- P_6: There is a need to estimate essential 'parameters' and test (the goodness of fit of) models.

While these were straightforward to delineate, mobilizing them is another matter.[1] The utility and practicality of these principles will be examined (as a secondary goal) in the context of empirically studying balance theory.

3. Structural balance theory

Heider (1946, 1958) has been credited with the first systematic statement of balance theory. It started life in his formulation as a process that operates within the minds of actors. The straightforward intuitions of his theory are captured in the triples shown in Fig. 1.

In this figure, $\{p, o, q\}$ represent three social actors and a social relation that is signed with positive and negative affect. My concern is focussed on relations among social actors and these actors' perceptions of the set of signed social relations within which they are located.[2] In these triples, $p \to o$ represents p's liking or disliking of o. Similarly, $p \to q$ represents p's signed tie to q. The tie $o \to q$ is p's *perception* of the signed tie from o to q. There is no requirement that this perception is veridical. The solid lines represent positive

[1] It may also be appropriate to join the Stokman and Doreian observations outlined before the list of principles into a principle in its own right.

[2] In this discussion, I am ignoring Heider's distinction between an affect relation and the 'unit formation' relation—consistent with most discussions of balance theory.

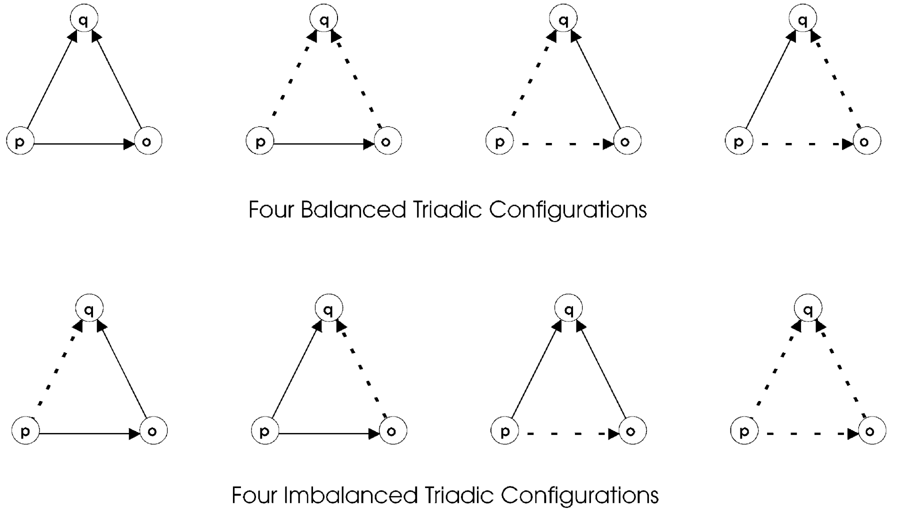

Fig. 1. Balanced and imbalanced signed triples.

ties while the dashed lines represent negative ties. Heider argued that some of the triples in Fig. 1 are 'comfortable' while others are not.

All of the triples in the top row of Fig. 1 are 'balanced' and 'comfortable'. The top left triple is a balanced configuration that is captured by the folk aphorism that "a friend (q) of a friend (o) is a friend (of p)". In the second triple from the left in the top row of Fig. 1, p likes o, dislikes q and perceives that o also dislikes q. This triple is captured by "an enemy (q) of a friend (o) is an enemy (of p)". In the next triple in the top row, p dislikes both o and q and perceives that o likes q. Heider viewed this as balanced and the corresponding folk aphorism is "a friend (q) of an enemy (o) is an enemy (of p)". The fourth balanced triple of the top row corresponds to "an enemy (q) of an enemy (o) is a friend (of p)". In contrast, all of the triples in the bottom row of Fig. 1 were viewed as 'imbalanced' and 'uncomfortable'. The leftmost triple represents a situation where p likes o while disliking q. Also, p perceives that o likes q. Heider viewed this as psychologically difficult for p. If p decided that she dislikes o, this triple moves from an imbalanced state to a balanced state (the third triple from the left in the top row of Fig. 1). Alternatively, p could decide to like q and so create an all positive (balanced) triple. It is possible also for p to change the perception of the tie from o to q. In the second triple (from the left) p likes both o and q but perceives (or knows) that o does not like q. This too was thought to be imbalanced for p—p's two friends do not like each other—and so uncomfortable. Two avenues for p achieving balance take the form of changing either the tie to o or changing the tie to q. Again, changing the perception of the o to q tie to a perceived positive tie achieves balance. The third triple in the bottom row can be given a similar interpretation. The rightmost triple is rather awkward but was treated as imbalanced by Heider. In Heider's formulation, imbalance creates discomfort and this generates 'forces' that move triples towards balance. Of course, these triples can be viewed simultaneously from o's or q's vantage points. And the group situation gets more complicated when p, o, and q are embedded in many other triples subject to the same dynamics. This more general configuration was treated by Cartwright and Harary (1956) when they generalized structural balance theory using signed graphs. More formally, a *signed network* is an ordered pair, $(\boldsymbol{G}, \sigma)$, where:

1. $\boldsymbol{G} = (\mathcal{V}, \mathcal{A})$ is a digraph, without loops, having a set of vertices, $\mathcal{V}$, and a set of arcs, $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$; and
2. $\sigma\colon \mathcal{A} \to \{P, N\}$ is a sign function. The arcs with the sign P are *positive* while the arcs with the sign N are *negative*. Equivalently, and consistent with most diagrams of signed networks; $\sigma\colon \mathcal{A} \to \{+1, -1\}$.

Such a social network can be denoted by $(\mathcal{V}, \mathcal{A}, \sigma)$ where v_i is an actor and an ordered pair $(v_i, v_j) \in \mathcal{A}$ is a tie from the actor to the actor v_j if $v_i, v_j \in \mathcal{V}$. The actors $\{v_i\}$ in this formulation include $\{p, o, q\}$. For the *poq* triples [3] of Heider (1946, 1958) and of Newcomb (1961), $p, o, q \in \mathcal{V}$ while the ties are elements of $\mathcal{A}$. In the formulation of Cartwright and Harary (1956), the networks can have any size. Their key step was to define the sign of a triple as the product of the signs of the links in the triple. If the resulting sign is positive the triple is balanced and if this sign is negative, the triple is imbalanced. Consistent with this, the sign of each triple in the first row of Fig. 1 is positive and the sign of each triple in the bottom row is negative. The idea of the sign of a triple extends naturally to the sign of a semi-cycle of any length. [4] A network (graph) is balanced if all of its semi-cycles are balanced. Using this formulation, they produced a remarkable theorem concerning signed graphs:

Theorem 1. *A signed graph $(\boldsymbol{G}, \sigma)$ is balanced if and only if the set of vertices $\mathcal{V}$ can be partitioned into two subsets so that every positive arc joins vertices of the same subset and every negative arc joins vertices of different subsets.*

This will be referred to as the 'first structure theorem'. Its form is a prime example of using mathematics to reveal a deeply significant structural feature of *groups* given the presumed balance processes operating in the minds of actors. It links the micro-level processes of actors to the macro-structure of groups generated by balance theoretic processes. If balance theoretic processes are operative, the resulting structure is a group where there are two mutually hostile subgroups each with internal solidarity. [5]

Although the conceptual standing of the all negative triple was unambiguous in Heider's account, having it imbalanced is less than satisfactory. Davis (1967) examined the consequences of specifying this all negative triple as balanced. He generalized the first structure theorem into the 'second structure theorem':

Theorem 2. *A signed graph $(\boldsymbol{G}, \sigma)$ is k-balanced for $k \geq 2$ if and only if the set of $\mathcal{V}$ can be partitioned into k subsets, called plus-sets, so that every positive arc joins vertices of the same subset and every negative arc joins vertices of different subsets.*

[3] Strictly, Heider and Newcomb used the *pox* triples where x can be any social object. However, my concern here is with social actors and q replaces x in this discussion.

[4] If the graph is complete, it is enough to examine just the triples.

[5] The structure theorem holds trivially if there is just one group and all of the relations are positive.

At least two lines of empirical inquiry were triggered by structural balance theory. One stayed close to the roots of the theory and examined triples and perceptions of signed triples (usually in simple laboratory experiments). The other ventured out into the realm of groups that were larger than triples. It is this stream of research and the study of the structure of signed networks in groups that concerns me here. Virtually all of the data used to study structural balance in real groups took the form of examining *relations among the actors* of the group. As a result, the accumulated empirical literature on structural balance does *not appear to speak directly* to Heider's initial formulation. By looking only at signed ties between actors, the idea that there are mechanisms operating within the minds of the actors was discarded. If mechanisms were operating they were assumed to operate at the *level of the group*. This is tantamount to assuming that the signed network structure changes in a self-organizing fashion regardless of the attributes of the actors. Although this is a reasonable specification, making it handicaps us in the study of the dynamics of structural balance.

The central substantive insight of structural balance theory is that signed networks move towards balance over time. This is the so-called *fundamental* structural balance theoretic *hypothesis* (FSBH). To assess this hypothesis requires two things: (1) longitudinal data and (2) a measure of imbalance. There has been a dearth of longitudinal signed network data. Analyses of two of the few longitudinal data sets are considered in Section 5.

There are at least two approaches to measuring the amount of imbalance in a graph. One approach is to get a census of semi-cycles and construct ratios of the number of balanced semi-cycles to the total number of semi-cycles. This approach bogged down because it was not clear how to treat semi-cycles of differing lengths and determining all cycles is a difficult graph theoretic problem. A good general algorithm to locate cycles is a recent development (Hummon and Fararo, 1995). An alternative approach is to adopt Harary's (1959) 'line index' of imbalance as the number of lines for which the reversal of their signs leads to a balanced network (graph). Numerically, this index is the same as the number of lines whose deletion leads to a balanced graph (Harary et al., 1965, p. 350).

While having a measure of imbalance makes it possible to discern movement towards balance (or not) it does nothing to describe the structure of the group through time. The kinds of structures described in the structure theorems (or approximations to those structures) may be of even more sociological interest than just the movement of an imbalance measure through time. Doreian and Mrvar (1996) proposed a method for both locating partitions of actors so that the pattern of signed ties is closest to the 'ideal' types of structures as described by the structure theorems and a measure of imbalance that was the line index (negation or deletion) proposed by Harary (1959). More formally, they sought to determine the clustering(s) $\boldsymbol{C}^*$ for which

$$P(\boldsymbol{C}^*) = \min_{\boldsymbol{C}\in\Phi} P(\boldsymbol{C})$$

where $\boldsymbol{C}$ is the clustering of a given *set of vertices* $\mathcal{V}$, Φ the set of all possible clusterings and $P\colon \Phi \to \mathbb{R}$ a *criterion function*.

The criterion function is constructed from inconsistencies with a balanced structure. These inconsistencies take one of the two forms: they are either negative ties within a plus-set or positive ties between plus-sets. A criterion function can be based on this idea.

Letting $\mathcal{N}$ be the total number of negative ties within plus-sets and $\mathcal{P}$ be the total number of positive ties between plus-sets, the criterion function is defined as:

$$P(\boldsymbol{C}) = \mathcal{N} + \mathcal{P}$$

In this formulation, the two types of inconsistencies are treated as equally important: the criterion function is simply the count of all inconsistencies regardless of their type.[6]

The criterion function is then minimized by using a relocation algorithm. The procedure provides sets of empirical partitioned structures that are as close to an exactly balanced partition as is possible. In addition, for each such partition, their method provides a list of ties that are inconsistent with balance. One of the few (and legendary) longitudinal data sets with signed data was collected by Sampson (1968). Doreian and Mrvar used the Sampson data set to test the FSBH described.

These results are considered briefly in Section 5. The Sampson data come in the form of reported signed ties between actors and have a variety of weaknesses. In terms of my discussion here, one important feature of those data is that they are defined at the group level and are utterly silent about what could have been going on in the minds of the actors. In short, Heider's defining formulation of cognitive and affective processes going on in the minds of actors was ignored (in common with all other studies at the group level of signed social networks). This may have been an important omission in so far as it ignores the first of the principles, P_1, outlined by Stokman and Doreian (1997) and reproduced in Section 2.

4. Causality and causal mechanisms

Doreian (2001b) provides a discussion of causality with regard to social network analysis. In that discussion, four possible ways of conceptualizing causality are distinguished: (1) system causality; (2) statistical (or predictive) causality; (3) mechanism causality; (4) algorithmic causality. While attention is confined here primarily to the second two conceptualizations of causality, the first has a role to play in this discussion.

4.1. System causality

Social network analysts view their enterprise as having a structural focus that is concerned with the structures of social networks, their effects and how they are generated. The traditional social scientific language of variables and relations between variables does not fit neatly into this paradigm (Wellman, 1988; Abell, 1987). Yet this is a natural language in which system causality can be expressed. Provisionally, a social system can be expressed

[6] A slightly more general criterion function is:

$$P(\boldsymbol{C}) = \alpha\mathcal{N} + (1-\alpha)\mathcal{P}$$

where $0 \le \alpha \le 1$. With $\alpha = 0.5$, the two inconsistencies are equally weighted. For $0 \le \alpha < 0.5$, positive errors (lines between plus-sets) are more important and for $0.5 < \alpha \le 1$, the negative errors (lines within plus-sets) are considered as more consequential.

in terms of a set of variables and one or more social processes linking and generating trajectories of these variables through time. More precisely, "the *state* (of the system) is some compact representation of the past activity of the system complete enough to allow us to predict, on the basis of the inputs, exactly what the outputs will be, and also update the state itself" (Padulo and Arbib, 1974, p. 21).

Let $\boldsymbol{x}$ represent a set of, say m, variables that characterize a system and let $\boldsymbol{x}(t)$ represent the state of the system at time, t. Suppose the system exists at time t_0 and receives inputs, represented by z. If the state of the system at t_0 is $\boldsymbol{x}(t_0)$ then the new state of the system is given by $\boldsymbol{x}(t_1) = \phi(t_0, t_1, \boldsymbol{x}(t_0), z)$ for some well defined function ϕ. "This function ϕ is called the *state transition map* and it tells us that if we specify two times t_1 and t_0, a state $\tilde{x}$, and an admissible input function z, then, if we start the system in state $\tilde{x}$ at time t_0 and apply the input function z, the system will end up at state $\phi(t_0, t_1, \tilde{x}, z)$ at time t_1" (Padulo and Arbib, 1974, p. 27). The output of the operation of the system is given by some function $\boldsymbol{y}(t_1) = \eta(t_1, t_0, \boldsymbol{x}(t_1), z)$. This characterization is for a deterministic system. If the subsequent state of the system and the output are not predicted exactly then the system is said to be stochastic. If $\boldsymbol{z}(t)$ is a time varying vector of exogenous variables, then a differential equation systems can be represented simply as:

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), \boldsymbol{z}(t), \boldsymbol{c}) \tag{1}$$

where $\boldsymbol{c}$ is a matrix of parameters (or products of parameters) and f is a well defined function. In general, such a differential equation system can be solved to yield

$$\boldsymbol{x}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{z}(t), \boldsymbol{c}) \tag{2}$$

where $\boldsymbol{g}$ is another well defined function. If there are no exogenous inputs, then Eqs. (1) and (2) are written as $\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), c)$ and $\boldsymbol{x}(t) = g(\boldsymbol{x}(t), c)$, respectively. Notationally, and conceptually, these two equations are those used by Fararo (1989, pp. 74–75) to represent a general dynamical system. This can be made more complex, and perhaps more veridical, through the use of stochastic differential equations or partial differential equations.

Eqs. (1) and (2) (or their variants without exogenous inputs) are particularly important as they describe the *generators* of the processes. If the parameters, the initial conditions (at t_0) and the exogenous inputs, $\boldsymbol{z}(t)$, are all known, these equations can be used to generate the states of the system and its outputs at each point in time. Fararo (1989) uses the term *recursive generation* to label such processes.

To bring this kind of thinking into social network analysis is not a straightforward task. Relations between actors can be expressed in terms of variables. In the case of signed relations, one variable could be the sign of relations which may switch through time. Another could be the intensity of the relation that can vary through time. Specifying the differential equations as generating mechanism will be tricky for two reasons.

First, the formulation in terms of a system represented by $\boldsymbol{x}(t)$ and $\boldsymbol{z}(t)$ is natural in the literature of systems theory where a distinction is made between a *system* and an *environment* within which the system is located. Clearly a social network is a (social) system and much of the (verbal) language used to describe networks stresses the interdependence of the parts of a network linked through the social ties of the social relation(s). While this 'system and environment' language is clearly applicable to conventional network imagery, it requires an orientation within which networks and their environments are both considered. Yet, network analysts tend to focus on 'the networks' of interest and then study these networks as objects divorced from the environments within which they are located. Of course, adopting a system causality perspective can be done for a network by itself but it would seem much more fruitful to include consideration its environment(s). With this change in orientation, the second difficulty to overcome is the specification of the equations themselves. Lacking a tradition where environments of networks are routinely included in the analysis of networks leaves network analysts handicapped in attempting to make these specifications.

4.2. *Statistical or predictive causality*

The literature on 'causal modeling' in the social sciences is huge but tangential for the current discussion. I have in mind all of the statistical tools that can be employed to detect, describe and explore causal relations *between variables*. Regardless of where the arguments are made in terms of regression equations or structural equation models as systems of equations, the difficulties are immense. See, for example, Clogg and Haritou (1997), Doreian (2001b), Freedman (1997). While equations, and systems of equations, can be estimated the causal modeling enterprise boils down to telling 'causal stories' (Morgan, 1997) given the estimated equations. This, in itself, is modest and most appropriate. Perhaps the telling of these stories provides a better understanding of the phenomena than do the claims of having 'detected' causal relations embodied in systems of equations. In the context of structural balance, it is not clear—at least to me—what the equations would look like.

Statistical methods are used for network analysis in a variety of other ways. A rich tradition starting with the Holland and Leinhardt (1981)family of p_1 models, leading to the Feinberg and Wasserman's (1981) application of log linear modeling tools, to Wasserman and Pattison's (1996) use of logit modeling methods to estimate p^* models provide clear evidence of the use of statistical procedures to analyze social networks. See Robbins and Pattison (2001) and Robbins et al. (2001) for recent applications of this approach. As far as I know, p^* models have not been applied to signed networks.

4.3. *Mechanism causality and sequences of events*

Looking at 'social mechanisms' and 'sequences of events' provides another approach to the issues raised by using causality in providing explanations of social organization generally and the generation of structure in the sense of social network structure. One link between predictive causality and this discussion is provided by Cartwright (1997) in her discussion of a causal structure conceived as "an ordered pair, $\langle V, E \rangle$, where V is a set of variables, and E is a set of ordered pairs of V where $\langle V, E \rangle$ is in E if and only if X is a direct cause of Y relative to V". She writes "(a) lternatively, V can be a set of events. But we should not be misled into thinking we are talking about specific events occurring at particular times and places" (1997, p. 343). Clearly, in Cartwright's formulation, events can be viewed as 'generic' or 'universal' in some broad sense that is divorced from specific empirical (concrete) locations in time. Presumably generic processes can be described in terms of these general mechanisms. However, it is impossible to think in terms of generic mechanisms without the observation of specific events and event sequences having occurred at some specific times and places.

A focus on ‘specific events’ draws us into the arena of ‘social mechanisms’, a concept not defined uniquely in the literature. Even so, the general idea is clear as well as the role of mechanisms in constructing social theory. For my purposes here, I will use the definition put forward by Hedström and Swedberg (1998, p. 1) when they claim that the theory goal is “to explicate the social mechanisms that generate and explain observed associations with events”. They echo Cartwright’s caution by stating that a social mechanisms approach “should not be confused with a purely descriptive approach that seeks to account for the unique chain of events that lead from one situation or event to another.” This is a very subtle distinction: we observe sequences in the empirical world and seek to account for them in terms of ‘generalized’ mechanisms that are also sequences of some kinds of events. Hedström and Swedberg (1998, p. 2) insist that the distinction be made as they go on to observe that their vision for explanatory sociology includes the creation of “an ensemble of such fundamental mechanisms that can be used for explanatory purposes”. For them, a simple description of a sequence of events in a specific empirical context is not an explanatory account. This seems too extreme.

A useful example comes from studies of animal social structure in the form of dominance hierarchies. Chase (1992) and Fararo et al. (1994) consider ‘fighting’ and ‘bystander’ mechanisms as generators of dominance hierarchies among primates (among other animal species). The outcome of repeated fights establishes dominance patterns between primates in the pairs of primates (in a particular primate group) who fight each other. As this has been observed in many primate groups, it seems reasonable to talk of a generalized fighting mechanism as a generator of dominance. However, this mechanism, by itself, is insufficient as an account of the generation of the observed hierarchies in primate groups. The (specific) fights and their outcomes are observed by other primates in specific groups. These bystanders develop different orientations towards the ‘winners’ (an increased likelihood of being submissive) and the ‘vanquished’ (an increased likelihood of trying to be dominant). The arguments involve a subtle alternation between specific event sequences in particular primate groups and conceptions of generalized mechanisms that hold for all primate groups (satisfying the requisite scope conditions) in order to have a general theory about dominance in primate groups.

This discussion points to the task of constructing general descriptions of mechanisms and event sequences that generate signed network structures through time. Mindful of the distinction made by Cartwright and by Hedström and Swedberg, this will entail both specific empirical event sequences and generalized sequences and mechanisms. However, it starts with a consideration of specific event sequences.

4.4. Algorithmic causality

Algorithmic causality takes its meaning from the term algorithm. “Informally, an algorithm is any well defined computational procedure that takes some value, or a set of values, as input and produces some value, or set of values, as output” Cormen et al. (1990, p. 1). Of course, this is very similar to the definition of system causality. However, the crucial distinction is that the language of differential equations is dropped from the definition and emphasis is placed on computational rules and computational processes. The key idea is the embodiment of rules in the code of the algorithm. For this discussion, I am concerned with the coupling of substance with the specification of the rules (as described by Hummon and

Fararo (1995)). In this context, object-oriented programming and parallel processing can be used to represent rich social processes via rules operating on actors, events and relations. “The rules can represent activities whose precise order is not determined in the simulation, yet the global outcomes depend on the order in which the activities occur” (Doreian, 2001b, p. 99). These ideas are mobilized in a simulation study of Hummon and Doreian (2002) that is described briefly in Section 6.

5. Empirical assessments of balance theory

As noted in Section 3, any attempt to empirically assess the FSBH requires longitudinal data on signed relations for a social group. Doreian and Mrvar (1996) used the Sampson data as one of the data sets in which they introduced their algorithm for partitioning signed networks into plus-sets where the partitioned structure is as close as possible to perfect balance. For each of three successive time points, the group of trainee monks in the monastery studied by Sampson was partitioned into three meaningful subgroups (that were richly described in Sampson’s (1968) ethnography) where the number of inconsistencies with exact balance were 35, 22 and 21 when the original valued ties are considered. When only the signs of binary ties are considered, the partitions are the same and the number of inconsistencies is 20, 14 and 11 for the three time points. This provides mild support for the FSBH with the additional interesting finding that the number of positive inconsistencies (positive ties between plus-sets) outnumbered the number of negative inconsistencies (negative ties within plus-sets). Intuitively, this makes sense as the positive ties can be viewed as idiosyncratic and not too deviant [7] while the negative inconsistencies threaten the internal solidarity within a plus-set. This finding has been repeated in virtually all partitions of empirical signed networks that I have seen.

Doreian et al. (1996) provided a more extensive examination of the FSBH using the Newcomb’s (1961) data described by Nordlie (1958). These data came from a study of pseudo-fraternity where 17 previously unacquainted young men were given room and board in a ‘fraternity’ in exchange for providing data including weekly sociometric ratings of each other over a 15-week period. Their central findings were three-fold. Reciprocity levels were significantly higher than would be expected by chance at week 1 and, with minor oscillations, reciprocity remained at the same level throughout the study period. Transitivity patterns were quite different. The amount of transitivity was close to 0 and did not exceed what would be expected by chance at the outset but it started to climb immediately. By week 3, it reached a level that was significantly different from what would have been expected by chance and it continued to climb until reaching a plateau at week 8. It did not increase much beyond that point. The amount of imbalance as measured by the line index for the optimal partitions located by the method of Doreian and Mrvar (1996) dropped throughout the first 14 weeks of the study period—but with some reversals, see Table 6.

These are important results because they show that reciprocity, transitivity and balance are network processes that *have different time scales*. Interesting as these results are, they do not tell us how to build a general model of the evolution of a signed social network

[7] Although *Romeo and Juliet* provides suggestive evidence to counter this.

that incorporates and couples all three processes with their different time scales. At face value, some sort of system causality model could be specified. However, the difficulties seem rather daunting.

Another result from this empirical examination was the evolution of the partition structure through time. Table 1 shows the partition structure at the first and final time points. At the beginning, there are three clusters of sizes 4, 5 and 8. In contrast, at the final time point there is one large subgroup of size 13 with the remaining three clusters being two singletons and a dyad. The *structure* of the group had *changed dramatically*.

Table 1
First and final partition structures for the Newcomb data

Partition into Three Clusters at First Time Point																	
	A	F	H	M	B	D	G	N	P	C	E	I	J	K	L	O	Q
A	0	1	-1	1	0	0	0	0	0	0	0	-1	-1	1	0	0	1
F	0	0	1	1	0	1	0	0	0	0	-1	1	-1	-1	0	0	0
H	0	1	0	1	0	0	-1	-1	0	-1	0	0	1	1	0	0	0
M	1	1	-1	0	-1	0	0	0	0	-1	1	0	0	0	0	1	0
B	0	0	-1	0	0	1	1	0	1	-1	0	0	0	-1	0	0	1
D	0	1	-1	0	1	0	1	0	0	-1	-1	0	0	0	0	0	1
G	-1	0	0	-1	1	1	0	0	0	0	-1	0	0	0	1	0	1
N	-1	0	-1	-1	0	0	1	0	0	0	0	1	1	0	0	1	0
P	0	-1	0	0	0	1	-1	0	0	-1	0	1	0	1	0	0	1
C	0	0	-1	-1	0	0	0	0	-1	0	0	0	0	1	1	1	1
E	-1	-1	1	0	0	0	0	-1	0	0	0	0	0	1	1	0	1
I	0	0	-1	0	-1	-1	1	0	0	0	0	0	0	1	1	0	1
J	1	1	0	0	-1	-1	1	0	0	0	0	0	0	-1	0	1	0
K	0	-1	-1	0	0	0	0	-1	0	1	0	1	0	0	1	0	1
L	-1	-1	0	-1	0	0	0	0	0	1	0	0	1	1	0	0	1
O	-1	0	0	0	0	0	0	-1	-1	1	1	0	1	1	0	0	0
Q	0	0	0	0	-1	1	0	-1	0	0	1	1	0	0	1	-1	0

Four Cluster Partition at Final Time Point																	
	A	B	D	E	F	G	H	I	K	L	M	N	Q	C	J	O	P
A	0	0	0	0	0	0	1	0	0	0	1	1	1	-1	-1	0	-1
B	0	0	1	1	0	0	0	0	0	1	0	0	1	0	-1	-1	-1
D	0	1	0	1	1	0	0	0	0	0	0	-1	1	-1	-1	0	0
E	0	1	1	0	0	0	0	1	0	0	1	0	0	0	-1	-1	-1
F	0	0	1	0	0	0	1	1	0	0	1	0	0	-1	-1	0	-1
G	0	1	0	-1	0	0	0	0	1	1	0	0	1	0	-1	0	-1
H	1	0	1	0	1	0	0	0	0	0	1	0	0	-1	-1	0	-1
I	0	0	1	0	1	0	0	0	0	0	0	1	1	0	-1	-1	-1
K	0	1	1	0	0	0	-1	0	0	1	0	0	1	0	-1	0	-1
L	0	1	0	0	-1	1	0	1	0	0	0	0	1	0	-1	-1	0
M	1	0	0	0	0	1	1	0	0	0	0	0	1	-1	-1	0	-1
N	1	0	0	-1	1	0	0	1	1	0	0	0	0	-1	-1	0	0
Q	1	1	1	0	0	0	0	1	0	0	0	0	0	-1	-1	0	-1
C	0	0	0	0	1	0	0	1	0	1	0	-1	1	0	-1	0	-1
J	1	0	0	0	0	1	1	0	0	1	-1	0	0	-1	0	0	-1
O	0	0	1	-1	0	0	0	1	-1	0	0	0	1	0	-1	0	1
P	0	0	1	0	0	0	-1	1	0	1	0	0	1	-1	-1	0	0

Table 2
Pre-transitive conditions, balance and evidence

Triple	Tie 1 ($p \rightarrow o$)	Tie 2 ($o \rightarrow q$)	Tie 3 ($p \rightarrow q$)	Balance	Supports balance
PPP	+	+	+	Yes	Supports
PPN	+	+	−	No	Supports
PNP	+	−	+	No	Supports
PNN	+	−	−	Yes	Supports
NPP	−	+	+	No	Contradicts
NPN	−	+	−	Yes	Contradicts
NNP	−	−	+	Yes	Contradicts
NNN	−	−	−	Yes/no	(Supports)

Doreian and Krackhardt (2001) returned to the Newcomb data with concerns that focussed on the FSBH and the idea that ‘the’ balance *process* might not be a *single* process. They focused in their analysis on the eight triads of Fig. 1 and considered ‘pre-transitive conditions’ (because the data come in the form of the sociometric ratings, the $o \rightarrow q$ tie is the tie from o to q and not p’s perception of that tie). The pre-transitive condition is formed by the $p \rightarrow o$ and the $o \rightarrow q$ ties with their signs and the question is whether of not the $p \rightarrow q$ tie completes the triple in ways that are consistent with structural balance. Their design is shown in Table 2 where P denotes a positive tie and N a negative tie. In the first column, the triples are listed in terms of their signed ties. Tie 1 and Tie 2 form the pre-transitive conditions and Tie 3 is the tie completing the triple. The next column indicates whether the triple is balanced or imbalanced. The only ambiguity is in the last row of the table. In the Heiderian view this triple in imbalanced while in the Davis view it is balanced.

The FSBH states that the signed structure moves towards balance through time. In the Doreian and Krackhardt analysis this was re-expressed in terms of the eight triples shown in Fig. 1 (and Table 2):

- H_1: Through time, the proportion of balanced triples increases.
- H_2: Through time, the proportion of imbalanced triples decreases.

Doreian and Krackhardt counted the occurrences of these triples through time and used permutation tests to determine if they occurred more frequently than would be expected under random conditions (given the number of positive and negative ties in the network at each time point). Some triples were present or absent at levels that would not be expected under randomness at the outset and by week 6 all triple types were occurring more often or less often (expressed in terms of relative frequencies) than would be expected by chance. The final column of Table 2 shows the results in terms of the sub-hypotheses. If balanced triples occur more frequently through time and if imbalanced triples occur less frequently through time, the FSBH is supported. As described in Table 2, the triple is p-centric where $p \rightarrow o$ and $p \rightarrow q$ are relations involving p and $o \rightarrow q$ is the third relation. The actors (p, o, q) are in a q-centric triple as well as an o-centric triple and the ‘counting of triples’ was done for all permutations of the actors in the network.

Overall, the evidence in Table 2 is mixed. The *PPP* and the *PNN* triples are balanced and they do occur more frequently through time.[8] However, the *NPN* and the *NNP* triple are both balanced and they occurred *less* frequently through time. When we turn our attention to the imbalanced triples we see that the *PPN* and *PNP* triples do occur less frequently through time, consistent with the FSBH. However, the imbalanced *NPP* triple occurs more frequently through time and contradicts the FSBH. The all negative *NNN* triple provides ambiguous evidence. These triples become more frequent and contradict the FSBH under Heider's formulation. However, for the Davis formulation, the increasing relative frequency of this triple supports the FSBH.

At a minimum, the FSBH has to be qualified and it seems best to break it into sub-hypotheses. The singular balance process can be broken into sub-processes and only *some* of them seem to operate in ways that are consistent with balance theory. This, however, is not the end of the story as the results shown in Table 2 can be interpreted in ways that suggest there are rival hypotheses to consider or rival mechanisms that may operate as a part of the structural dynamics.

Staying within the Heiderian perspective, the triples that behave in accordance with the FSBH all have the first tie ($p \rightarrow o$) of the triple positive. This suggests that the mechanism operates only when that tie is positive. Whenever $p \rightarrow o$ is negative, there is no balance mechanism at work. This result was, in fact, anticipated by Newcomb (1968).

A second way in which Doreian and Krackhardt looked at these results was to see if there was anything in common with the triples whose relative frequency increased through time. These triples are *PPP*, *NPP*, *PNN* and *NNN*. The common feature they share is that the signs of the last two ties are the same. That is, regardless of the sign of the tie between p and o, they 'agree' with each other about q. This suggests a non-structural mechanism: the attributes of q are critical in the formation of signed ties with q. However, this can be qualified by a third interpretation suggested by Doreian and Krackhardt, one that I take up in Section 7. Another possible interpretation is that when p and o are linked by a negative tie, they are motivated to compete as rivals for the attention of q and this can accounts for the imbalanced *NPP* triple increasing in relative frequency through time.

These empirical analyses suggest looking at structural balance theory also in light of the principles suggested by Stokman and Doreian (1997). It seems these principles are left implicit when balance theory has been considered at the group level in empirical studies. A case can be made that by ignoring actor level processes, nothing is assumed about actors having goals and, as a result, P_1 is ignored. Alternatively, an implicit assumption could have been made that the actors are motivated to reduce imbalance and it is this mechanism that drives the group level balance dynamics. Even so, it seems best to make such an implicit assumption explicit if it is relevant for understanding balance processes.

To the extent that only the observed ties have been seen as relevant for studying structural balance, it would seem that actors are not assumed to have local information in balance theoretic studies. As a result, P_2 has been ignored also. However, the assumption that actors have local knowledge is not inconsistent with balance theory and it is possible to argue that P_2 has been made implicitly. The main problem created by leaving the assumption implicit is that it is not possible to know the different amounts local knowledge held by

[8] The actual increase in the relative frequency of the *PPP* triple was quite modest.

actors nor how this differential knowledge enters balance theoretic processes. The Doreian and Krackhardt's (2001) result that the sign of $p \to o$ matters regarding the operation of balance mechanisms suggests an aspect of local knowledge that may be relevant. Further, in the Heider's (1946) formulation, the $o \to q$ tie in Fig. 1 is the *perception* of the tie by p. The 'real' $o \to q$ has been used in balance theoretic studies. This suggests that knowledge has been viewed as global and, most likely, accurate. Both assumptions seem problematic.

It appears that P_3 has not been considered and the assumption that actors act in parallel has not been made explicitly even though such an assumption is not inconsistent with balance theory. With what is left implicit, it is not possible to examine parallel action streams coupled into a model of group dynamics. Structural balance is certainly simple and is consistent with P_4. However, P_4 is trivial as stated because 'simple' is a relative term. The Doreian and Krackhardt results suggest that some movement from the simple balance model is warranted. There are multiple balance mechanisms that might be operating together with rival mechanisms. Incorporating these processes with mean less simple balance models.

If both actor level processes and a group level process are relevant, then P_5 is ignored and there are insufficient empirical referents in the usual study of structural balance at the group level. Finally, it seems that P_6 is honored most often in the breach. The FSBH is remarkably simple and empirical studies demonstrating the decline of imbalance have been hampered by not having a good semi-cycle based measures of imbalance (Hummon and Fararo, 1995). There has been no statistical assessment of whether movements of balance through time are statistically real. While the line index of balance used by Doreian and Mrvar (1996) is simple and provides a measure of fit, it also lacks a statistical foundation.

6. Simulating balance theoretic processes

Strong as the evidence—as described in the previous section—might be, the results are limited given the origins of structural balance theory. We know nothing of the actors [9] nor do we have any information concerning their cognitions and feelings. The same is true for the Sampson data and all data obtained from groups where only signed ties between actors have been recorded. The potent generalization of balance theory provided by Cartwright and Harary (1956) inspired an empirical approach where the cognitive dynamics were no longer relevant. This may have been costly for the long term 'health' of the theory they generalized.

I have no problem with the idea of a self-organizing balance theoretic process that operates at the group level. However, modeling a balance process *only* at this level ignores the Heiderian foundation and the P_1 principle of Stokman and Doreian (1997). It seems reasonable to attempt to incorporate cognitive dynamics into a broader theoretical model. Lacking empirical data about this, Hummon and Doreian (2002) built a simulation model that incorporated both a group level process and an actor level process for each actor.

[9] All descriptive information on, for example, who were roommates and common floor membership in the Newcomb data, have been lost.

They assumed that the FSBH is correct *at the level of the actor* and that states of imbalance are undesirable for actors. In the Stokman and Doreian imagery, a part of the goals of actors is to reduce imbalance, consistent with Heider's formulation (and P_1).

Hummon and Doreian (2002) coupled two simulation approaches. One was a multi-thread agent-based model that was designed to show graphically the behavior of the simulated actors. The second was a discrete event simulation model that was designed to implement Monte Carlo experiments in order to get a sense of the range and distribution of behaviors generated in the simulated processes. Both approaches implemented the same actor behavior where the actors behaved as separate units—consistent with P_3 as described in Section 2—while being tied through the ties they generated.

Each actor has an image of the signed group structure but this image need not be consistent with the 'real' signed structure. This step is consistent with P_2 of Stokman and Doreian (1997). The image held by an actor depends, in the simulation, on the information reaching that actor. Given the image an actor has, that actor strives to reduce imbalance. For each possible change of ties, an actor knows whether or not this change (by itself) will reduce imbalance. In the case where there are multiple equally good changes, one is selected at random. Each actor does this separately, consistent with P_3, and reports the change. The way an actor, say p, reports the change is a design variable of the simulation where there are four options: p reports the change only to the other actor, q, to whom the tie is changed; p reports the change only to actors linked to p by positive ties (tell friends); p reports the change only to those to which p is linked—which includes also negative ties—i.e. tell acquaintances; or tell every other actor in the group. In this way, information held by actors is local (in all but the last communication mode), consistent with P_2.

The transmitted information also goes into the 'group structure' or model. In the simulation, the Doreian and Mrvar procedure is used to locate those partitions at the group level that are as close to perfect balance as possible. If there are multiple such partitions, one is selected at random and the summary partition information is reported back to the individual actors. This is joined with the local knowledge of actors. The group level structure has the kind of information collected by Newcomb and Sampson. In this way, the group 'process' constrains and affects the actions of the individual actors. Feeding back the group structure in terms of balance and ties inconsistent with balance implicitly implements a group level process.

Additional design variables for the simulations include the size of the group and the incidence of negative ties at the first time point. The latter notion is used to capture the contentiousness of groups. All of these features are intended to provide plausible empirical referents as suggested in P_5. Throughout the simulations, the imbalance measure is computed as the number of ties that have to be changed in order to reach balance. The simulation continues until an equilibrium (that can be dynamic) is reached.[10] The value of $P(\boldsymbol{C}^*)$ is computed and forms a measure of fit—consistent with P_6. Only partitions whose values of the criterion function is the smallest are selected for further consideration by the actors.

In these simulations, communication mode, group size and contentiousness had complicated impacts of the amount of imbalance in the group structure, the number of actors

[10] Or time runs out with the process incomplete. In empirical contexts, time is a relevant constraint.

with balanced images, the number of acts needed to reach balance (or an equilibrium) and the average number of plus-sets at equilibrium. The details are not easily summarized and are not directly relevant for this discussion. However, the following results are of direct relevance (Doreian, 2001a):

1. there can be *very many* equilibrium outcome structures;
2. given the same initial conditions and the same generating rules, there are many outcome structures;
3. there were equilibrium group structures that were *not* balanced.

The first seems trivial at face value. However, it raises a key issue. In the simulated world, many instantiations of a generic process are possible while, in the empirical world, we are granted few realized instantiations of balance theoretic processes. Locating the (few) empirical results in the whole range of possible outcomes is difficult as we lack the knowledge of how diverse the outcomes can be. The second also seems trivial but it reinforces the first in so far as the *same process* can generate very *different* structural outcomes. [11]

These can range from a single group with only positive ties between actors to groups that are deeply riven into mutually hostile groups. The third outcome is, perhaps, the most interesting. Recall, the FSBH states that human signed structures move towards balance. The strange, rather embarrassing empirical result, across many studies, is that very few, if any, empirical signed networks among human actors reach a balanced configuration. Yet in the simulation outcomes where there are equilibrium *im*balanced structures, the individual actors had balanced images of the overall structure. At the actor level, if the perceived network is balanced, there is no 'discomfort' and no force impelling change. This leads to a plausible explanation for the seemingly embarrassing results (that groups do not reach balance) in a way that retrieves the FSBH by taking into account the cognitive dynamics of actors. Moreover, the explanation is faithful to the origins of the original theory. Further implications are considered in the next section.

7. Event sequences as generative mechanisms

Signed networks changed through time in the Sampson and Newcomb data sets as well as in the simulation. These changes can be viewed as sequences of events as specific ties change. As such, they are suggestive of more general mechanisms in the sense of Hedström and Swedberg (1998).

7.1. Sampson monastery data

The partitions of the Sampson data for each time point are shown in Table 3. Inconsistencies with balance are negative ties within plus-sets (diagonal blocks) and positive ties between plus-sets (off diagonal blocks).

[11] This differs from what would be expected in deterministic system causality models.

Table 3
Optimal balance partition of affect ties at three time points

T_2 Affect Ties

		1	2	7	12	14	15	16	3	13	17	18	4	5	6	8	9	10	11
John-Bosco	1	0	0	-1	0	1	0	0	1	0	0	0	0	1	-1	0	0	-1	0
Gregory	2	1	0	1	0	1	0	0	0	-1	-1	0	0	0	0	0	0	-1	0
Mark	7	0	1	0	0	0	0	1	0	0	0	0	-1	-1	-1	1	0	0	0
Winfrid	12	1	1	0	0	1	0	0	-1	0	0	0	-1	0	0	0	0	0	-1
Hugh	14	1	0	0	1	0	1	0	0	-1	-1	0	0	0	0	-1	0	0	1
Boniface	15	1	1	0	0	1	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0
Albert	16	1	1	1	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0
Basil	3	1	1	0	0	0	0	0	0	0	1	0	-1	0	0	-1	-1	0	0
Amand	13	0	-1	1	-1	0	0	0	0	0	0	1	0	1	-1	0	0	0	0
Elias	17	0	0	0	0	0	0	0	1	1	0	1	-1	-1	0	0	0	0	-1
Simplicius	18	1	1	1	0	0	0	-1	0	0	0	0	-1	0	-1	0	0	0	0
Peter	4	0	0	-1	0	0	0	0	-1	0	0	-1	0	1	1	0	0	1	0
Bonaventur	5	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1
Berthold	6	1	0	-1	-1	0	0	0	0	0	0	0	1	0	0	-1	1	0	0
Victor	8	1	1	0	0	-1	0	0	-1	0	-1	0	0	0	0	0	1	0	0
Ambrose	9	0	0	0	0	0	0	1	-1	0	-1	-1	0	1	0	1	0	0	0
Romuald	10	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0
Louis	11	0	0	0	0	1	0	0	-1	-1	-1	0	0	1	0	1	0	0	0

T_3 Affect Ties

		1	2	7	12	14	15	16	3	13	17	18	4	5	6	8	9	10	11
John-Bosco	1	0	1	0	0	0	0	0	0	0	0	0	-1	1	0	1	0	-1	-1
Gregory	2	1	0	1	1	0	1	0	0	-1	0	0	-1	0	0	-1	0	0	0
Mark	7	1	1	0	0	0	0	1	0	0	0	0	-1	0	-1	-1	0	0	0
Winfrid	12	1	1	0	0	1	0	0	-1	0	-1	0	0	0	-1	0	0	0	0
Hugh	14	1	1	0	1	0	1	0	0	0	-1	0	-1	0	0	0	0	-1	0
Boniface	15	1	1	0	0	1	0	0	-1	-1	-1	0	0	1	0	0	0	-1	0
Albert	16	0	1	1	1	0	0	0	-1	0	-1	0	-1	0	0	0	0	0	0
Basil	3	1	-1	0	0	0	0	0	0	0	1	1	0	0	-1	-1	0	0	0
Amand	13	0	-1	1	-1	0	0	0	0	0	0	1	0	1	-1	0	0	0	0
Elias	17	0	1	0	0	0	0	0	1	0	0	1	-1	0	-1	0	0	0	-1
Simplicius	18	0	1	0	0	0	0	0	0	1	1	0	-1	0	-1	0	0	0	-1
Peter	4	-1	-1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
Bonaventur	5	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
Berthold	6	1	0	0	-1	0	0	0	-1	0	0	0	1	0	0	-1	1	0	0
Victor	8	-1	-1	0	0	0	0	0	-1	0	0	0	1	0	1	0	0	1	0
Ambrose	9	0	0	0	1	0	0	0	-1	0	-1	-1	0	1	0	1	0	0	0
Romuald	10	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0
Louis	11	-1	-1	0	0	0	0	1	-1	0	0	0	1	1	0	0	0	0	0

T_4 Affect Ties

		1	2	7	12	14	15	16	3	13	17	18	4	5	6	8	9	10	11
John-Bosco	1	0	-1	-1	1	1	0	0	1	0	0	0	0	0	0	0	0	-1	0
Gregory	2	1	0	1	1	0	0	0	0	-1	0	0	-1	0	0	-1	0	0	0
Mark	7	0	1	0	1	0	0	1	0	0	0	0	-1	0	-1	-1	0	0	0
Winfrid	12	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hugh	14	1	0	0	1	0	1	0	0	0	-1	0	-1	0	0	-1	0	0	0
Boniface	15	0	1	1	1	0	0	0	-1	-1	0	0	-1	0	0	0	0	0	0
Albert	16	0	1	1	0	0	1	0	-1	0	-1	0	-1	0	0	0	0	0	0
Basil	3	1	-1	0	0	0	0	-1	0	1	1	1	-1	0	-1	0	0	0	0
Amand	13	0	-1	1	-1	0	0	0	0	0	0	1	0	1	-1	0	0	0	0
Elias	17	0	1	0	0	0	0	0	1	0	0	1	-1	0	-1	-1	0	0	0
Simplicius	18	0	1	0	0	0	0	0	1	0	1	0	-1	0	0	-1	0	-1	0
Peter	4	-1	-1	0	0	-1	0	0	0	0	0	0	0	1	1	0	0	0	1
Bonaventur	5	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1
Berthold	6	0	-1	-1	0	0	0	0	-1	0	-1	0	1	1	0	0	1	0	0
Victor	8	0	-1	0	0	-1	0	0	-1	0	0	0	1	0	1	0	1	0	0
Ambrose	9	0	0	0	1	0	0	0	-1	0	-1	-1	0	1	0	1	0	0	0
Romuald	10	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1	0	0
Louis	11	-1	-1	0	0	1	0	0	-1	0	0	0	0	1	0	1	0	0	0

The total count of inconsistencies through three time points[12] T_2, T_3 and T_4 are 20, 14 and 11, respectively. As noted, these figures are consistent with the FSBH and point to movement towards balance. However, *the change is neither simple nor smooth*. Counting null ties, there are 306 dyadic pairs. From T_2 to T_3, 85 ties (25%) change and 231 (75%) remain the same. There are 24 negative ties that change to a null tie. Of these changes only one (John-Bosco to Gregory) reduces imbalance while 23 make no difference. Of the 41 null ties that change, 19 change to a negative and make no difference to the amount of balance. Yet they can be viewed as making a difference as they are new negative ties between plus-sets (which is where the negative ties should be located according to balance theory). There are 22 null ties that change to positive ties. Of these, seven increase imbalance (introduce positive ties between plus-sets) and 15 make no difference to the line index of balance. However, these are new positive ties within plus-sets, which is exactly where they should be according to balance theory. There are 20 positive ties that change from T_2 to T_3. Of the four that become negative, three reduce imbalance (by removing -1 ties within plus-sets) and one that makes no difference. Of the 16 changes of positive ties to null ties, nine reduce imbalance while seven changes make no difference. It should be noted, however, that these are positive ties that are removed from within plus-sets. Overall, 20 of the 85 tie changes have an impact on the amount of imbalance while 65 changes make no difference to this index.

Another way of looking at the changes is to focus on the inconsistences with balance that are present at T_2. There are 20 inconsistencies. In the transition to T_3, 13 of these inconsistencies are removed while seven remain. However, another seven inconsistencies are created so that the total number of inconsistencies at T_3 is 14. The decrease in imbalance is a *net* change. Also, in terms of the line index, overwhelmingly, the changes that reduce balance are positive or negative ties becoming null. Seldom do ties change sign. Thus, it seems the line deletion version of the line index is more relevant than the line negation version.

For the transition from T_3 to T_4 there are only 61 ties (20%) that change. There are 15 negative ties that become null and while one change reduces imbalance, the other 14 do not. There are 13 null ties that become negative and while one of these changes increases imbalance, the remaining changes make no difference. Of the 15 null ties that become positive, two increase imbalance while 14 make no difference. There is one positive tie that becomes negative and it increases imbalance. Of the 16 positive ties that become null, six decrease imbalance and 10 make no difference. Overall, 50 of the tie changes from T_3 to T_4 have no impact directly on the amount of imbalance while seven changes reduce imbalance and four changes increase imbalance. These changes reduce the line index to 11, a net change of 3. All of the changes can be viewed as sequences of events with the complication that each actor in involved in many cycles and semi-cycles and these cycles may contain contradictory ‘forces’ impelling changes as actors strive to reach balance (if, of course, the FSBH is correct). Also, changes that do not have an impact on the line index of balance directly can operate to change the dynamics for actors. When positive ties are added within plus-set, they reinforce the internal cohesion within the plus-set even if their inclusion does not appear to affect the amount of imbalance. Of course, when positive ties are removed

[12] The labeling of these time points is taken from Sampson (1968). There was a T_1 where only some of the actors were present and a T_5 when many of the actors had left the monastery.

from plus-sets, they operate to reduce the internal cohesion of the plus-set. Similarly, when negative ties are added in ways that link actors in different plus-sets, they reinforce the balance processes while the removal of negative ties between plus-sets weakens them.

Why belabor this? Consider the three actors p, q, and o linked by ties that are shown in the lower panel of Fig. 1 (second triple from the left). Suppose further that each actor perceives this triple in the same way. In order to reach balance, p has two options: change the positive tie to o to a negative tie or change the positive tie to q to a negative tie. Each change, alone, will create a balanced triple (although the two triples will differ). Imagine the actor chooses the first option. Suppose that o also surveys the scene and decides to change the negative tie to q to a positive tie. If both changes are made, then the resulting triple is shown as the third triple on the left in the lower panel of Fig. 1. This new triple is *not balanced*—yet both actors acted in a way that was designed to achieve balance. Sequences like this suggest why balance need not be achieved quickly or directly. This gets more complicated when actors are located in many triples (and longer semi-cycles) simultaneously. Of course, both p and o could decide to make ties null and so create a disconnected triple, one that is 'vacuously' balanced.

The simple example is premised on the assumption that each actor perceives the triple in the same way. As described, tracking the changes is consistent with the P_3 principle of Stokman and Doreian (1997) where each actor is assumed (or allowed in a 'model') to act *separately*. Communication is assumed to be open and that each actor's perception is assumed to be veridical. However, given the simulation evidence, it is not hard to imagine how these assumptions can be false (and so invoke Stokman and Doreian's P_2 principle). Suppose p changes its positive tie to o to a negative tie but does not report this. The perception that p now has is the balanced triple that is second from the left in the top panel of Fig. 1. If o changes its negative tie to q into a positive tie then its image of the triple is the all positive (balanced) triple (first on the left in the top panel) of Fig. 1. If o remains silent about the change, then each actor's perception is a balanced triple. However, as noted, the two triples differ and the information of each actor is local. If a researcher collected data after the two changes, and the ties were reported accurately, the triple *in the data* would be the imbalanced triple that is third from the left in the lower panel of Fig. 1. With an 'inaccurate' triple recorded in the data and actors having different perceptions of the triple, at best, balance theoretic dynamics will be obscured. In the data, movement towards balance, if it exists, will not be direct. There will be changes that increase imbalance, changes that reduce imbalance and (many) changes that make no difference in the amount of measured imbalance.

7.2. Newcomb pseudo-fraternity data

The overall evidence for the Newcomb data suggests that imbalance decreased through most of the study period. However, this interpretation has to be qualified by the results of Doreian and Krackhardt (2001) with regard to the operation of multiple processes. Describing the event sequences in the Newcomb data is more complicated than in the Sampson data. First, the overall structure changes through time as shown in Table 4 where the letters label the actors as in Table 1. Each row reports a partition for each period. The partitions by period are unique for T_1–T_3 (as a four cluster partition), T_4, T_6 through T_{11}, T_{14} and T_{15}. For T_{12} and T_{13} there is a second optimal partition that has the same form as the unique partition of T_{15}.

Table 4
Optimal balance partitions of affect ties through time

T_1	A F H M	B D G N P	C E I J K L O Q	
T_2	A F H J M	B C D E G I K L N O Q	P	
T_3	A E F H J M	B C D G I K L N O Q	P	
T_4	A E F H M	B C D G I K L N P Q	J O	
T_5	A E F H M	B D G I K L N P Q	C	J O
T_6	A B D E F G H I L M N Q	C K	J	O P
T_7	A B D E F G H I K L M N Q	C	J	O P
T_8	A B D E F G H I K L M N Q	C	J	O P
T_9	A B D E F G H I K L M N Q	C	J	O P
T_{10}	A B D E F G H I K L M N Q	C	J	O P
T_{11}	A B D E F G H I K L M N Q	C	J	O P
T_{12}	A B D E F G H I K L M N Q	C	J	O P
T_{13}	A B D E F G H I K L M N Q	C	J	O P
T_{14}	A B D E F G H I K L M N Q	C O	J	P
T_{15}	A B D E F G H I K L M N Q	C	J	O P

At the first time point, T_1, the optimal partition has three clusters as shown in the top panel of Table 1 and in the first row of Table 4. For the next two time points, P has become a singleton in the presence of two large clusters. For T_4 and T_5, O and J form a cluster of size 2 and for T_5, C has become a singleton. At T_6 a large cluster is evident with two pairs of actors in distinct clusters plus J as a singleton. At T_7, the final structure has formed with the one large cluster. There are two singletons each in a cluster and a pair of actors in a cluster.

Not evident in Table 4 is the fate of these actors with regard to the receipt of positive and negative ties. This feature of change is shown in Table 5 (which is taken from Doreian and Krackhardt (2001)). The three actors (C, P, J) who do not belong to the large cluster receive many negative ties by the end of Newcomb's (1961) study period. They became heavily disliked actors. However, the interpretation that they possess attributes that make them disliked is not tenable when the receipt of positive ties is considered. At the first time point, J receives four positive ties and there are eight actors receiving fewer positive ties. Through time, J moved from receiving only two negative ties at T_2 to being universally disliked by the rest of the group members. Of course, it is possible that it takes time for people to discern the real attributes of others. But it seems more likely that as the structure of the group evolved into the form shown in Table 1, some of the attributes of the actors co-evolved with the changing group structure. Actor P also started by receiving two negative ties and then ended up receiving 12 negative ties. Another of the singletons in the evolved structure, C, started out with three positive ties and five negative ties. In the last week the count of negative ties received by C climbed to nine while his receipt of positive ties dropped to zero. These three most disliked actors received 37 negative ties (over 72% of the total of 51 negative ties). The receipt of negative ties is far more unequal than the receipt of positive ties at all time points and is the most unequal at the final time point. Changes in the distribution of the receipt of positive ties through time is far less dramatic.

Table 5
Receipt of positive and negative ties

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Negative ties															
A	5	4	4	3	2	1	1	0	0	1	1	0	0	0	0
B	4	4	1	3	4	1	0	0	1	0	1	0	1	0	0
C	5	5	4	5	5	6	9	10	10	9	9	9	9	8	9
D	2	1	1	1	0	1	0	0	0	1	0	0	0	0	0
E	3	1	0	2	1	1	1	0	1	1	1	0	0	2	3
F	4	4	4	4	4	3	1	3	3	3	1	1	2	1	1
G	2	4	5	4	2	0	0	0	1	0	1	1	2	1	0
H	8	7	8	8	6	2	1	1	0	3	2	4	0	2	2
I	1	1	2	0	1	1	1	1	2	0	0	0	0	0	0
J	2	8	8	11	12	13	13	15	14	11	13	11	13	14	16
K	3	1	0	2	0	2	1	1	1	1	0	2	2	2	1
L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M	4	0	1	1	2	2	3	2	0	0	1	1	1	1	1
N	5	4	4	1	0	0	2	1	2	2	0	1	1	1	2
O	1	1	2	1	3	6	6	5	5	7	9	9	7	6	4
P	2	6	7	5	10	12	12	12	11	12	12	14	13	13	12
Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Positive ties															
A	2	4	4	5	5	3	7	5	6	6	6	5	5	7	5
B	2	2	4	4	4	4	4	4	4	4	3	3	5	5	6
C	3	4	2	2	1	1	0	1	0	1	2	0	1	0	0
D	5	7	7	8	6	7	7	7	7	5	7	7	6	6	9
E	3	5	2	2	4	5	2	2	1	2	3	2	1	1	2
F	5	3	6	6	6	7	7	6	9	7	6	7	7	6	5
G	5	6	5	5	3	6	4	5	2	4	4	2	3	5	3
H	2	0	2	2	3	3	2	3	3	4	2	2	3	4	4
I	5	8	7	8	8	8	10	11	10	9	9	10	10	9	8
J	4	3	1	2	0	0	0	0	0	0	0	0	0	0	0
K	8	3	3	1	1	1	0	1	2	1	3	2	2	0	2
L	6	4	4	4	8	5	7	5	7	7	5	6	5	6	6
M	3	5	5	5	4	4	3	3	3	4	3	5	5	5	4
N	0	2	4	3	3	1	3	2	3	2	3	3	3	2	2
O	4	2	2	1	1	1	0	1	0	0	1	1	1	1	0
P	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1
Q	10	9	10	9	10	11	11	11	10	11	10	12	10	11	11

Another feature of the change through time is shown in Table 6 which reports the number of ties that change (including null ‘ties’) in each of the 14 transitions between successive time points. Also reported is the imbalance measure after each transition together with the net change of imbalance. There are many more tie changes than is reflected in the change in imbalance. While some tie changes reduce imbalance and other changes increase imbalance, most of the changes do not affect imbalance, as was the case with the Sampson data. Nor does the measure of imbalance monotonically decrease through time. In essence,

Table 6
Changes in ties and net imbalance

Transition	Ties changing	Net change imbalance	New imbalance
1	85	−2	28
2	50	−3	25
3	64	−1	24
4	50	2	26
5	66	−4	22
6	66	0	22
7	50	1	23
8	28	−1	22
9	42	1	23
10	48	−3	20
11	50	0	20
12	36	0	20
13	48	−1	19
14	46	1	20

the measure of imbalance stopped dropping at week 10 even though ties were changing. Clearly there were false starts and ‘mistakes’ in efforts to reduce imbalance. If some balance process mechanisms were inoperative and if there were other mechanisms at work, they confound the movement toward balance.

7.3. Further reflections on the simulation results

In contrast to the empirical situations where the actual mechanisms at work are unknown and efforts are made to identify some of them, in the simulations of Hummon and Doreian (2002), the mechanisms are known by design. Each actor strives to minimize imbalance and continues to do so until balance is reached or no further change towards balance is possible. As noted, there were outcomes where the ‘group structure’ was imbalanced while actor images of the group were balanced. Thus, having groups not reaching perfect balance can be consistent with the FSBH because Heider’s (1946) formulation concerned a mechanism inside the minds of actors as they sought to minimize ‘discomfort’.

Another outcome of the simulations was a by-product in the form of a transcript listing the *sequences* of acts as the balance dynamic unfolded. While we have not looked at them in a systematic fashion yet, it is evident that there are *very* long sequences of actions and that the number of actions increases with the size of the group. The increase is not linear with the group size and depends also in the initial contentiousness of the groups. One reasonable conclusion is that balance theory processes take a long time to play out as actors strive to reach balance. And this conclusion prompts the suggestion that most empirical studies of structural balance in social groups are too *short*.

Finally, the simulation of Hummon and Doreian (2002) was more of an exercise in theory that specified structural balance as a multiple-level process. Most empirical studies of structural balance simply forget the processes operating in the minds of actors. If balance does operate, it operates both in the minds of actors and at the group level. Having one process (or set of processes) without the other is an impediment to obtaining a general understanding of structural balance theoretic processes.

8. Discussion

This paper began, in the context of network evolution, with a pair of conflicting assessments of structural balance theory. Opp (1984) saw failure for the theory and a corresponding loss of interest by social psychologists and sociologists. Davis (1979) saw the theory as successful. Of course, the criteria used in the two assessments were different. For Opp, there was a long line of inconclusive evidence and contradictory results. For Davis, the formalization of balance theory provided the foundation for an elegant result that linked the micro-foundations of actor level processes to a group (macro-level) structure. The theory was plausible and there was some evidence supporting it. My argument here is that as far as Opp and Davis go, they are correct in their conclusions and, while at odds, these conclusions can be reconciled.

It is useful to think of structural balance as one (of many) approaches to the evolution of social networks. The ideas of Stokman and Doreian (1997) suggest principles whose inclusion in the study of balance theory have empirical payoff. When the actor level processes are excluded—as in both the Sampson and Newcomb studies—the study of balance theory is impoverished. True, leaving them out is consistent with P_3 as the model is certainly simple. However, such models are too simple and Stokman and Doreian suggest that less simple models be considered if the simple model fails or appears to fail. Implementing P_1 means that actor processes can no longer be left out safely if a more comprehensive understanding is sought. Actors have goals and it is reasonable to bring them into a model. The Hummon and Doreian's (2002) simulations suggest a way of doing this and suggest that it is necessary to incorporate actor-level processes.

The simulation results also suggest a way in which actors can come to have local information that is not veridical with respect to the group structure and how they may have images that are inconsistent with each other. The mechanism for creating this was communication. By having different modes of communication, the distribution of local information held by actors was changed. As a result, different actors came to have different amounts of both accurate and inaccurate local information. Social life may be more complicated. In the simulations each actor communicated in the same fashion (with a single run) and this made it possible to study the impact that communication modes have on the simulated outcomes. It is likely that different actors communicate in different ways. While this change will be incorporated into future simulations, the empirical study of balance with be more difficult if these 'style' differences have to be considered.

The results of Doreian and Krackhardt (2001) point to two serious problems in the empirical study of balance theory. One is that the 'single' balance theoretic process must be considered as a bundle of mini-balance processes (or mechanisms) and that not all of them need be operative in a given empirical context. The second difficulty is that multiple processes in addition to balance may be at work. Just as differential popularity may be confused with transitivity (Feld and Elmore, 1982) so can differential liking and disliking be confused with structural balance. While such considerations can be incorporated into the kind of simulations done by Hummon and Doreian, studying competing mechanisms empirically will be difficult. This is made more complicated, at least in principle, by the Doreian et al. (1996) results that reciprocity, transitivity and balance appear to have different time scales.

This last idea makes me pessimistic in the short run with regard to the construction of models in the form of systems of equations. This implies that we may not be ready, at this time, for the construction of adequate ‘system causality’ models. I suspect that the role of ‘statistical causal’ modeling is limited and may be confined to estimating system causality models once they are formulated. If we are not ready to build system causality models and estimate them, we have ‘mechanism causality’ models and the generation of sequences of events to consider together with ‘algorithmic causality’ (that was the core of the simulation models discussed).

Empirically, we are likely to be involved in the observation of sequences of events. The analyses of the Sampson and the Newcomb data sets, as discussed, were very limited. But they hint strongly that the sequences of events will not be coherent in the form of simple direct movements that are easily tracked and comprehended. Even within the simulations and the incorporation of only structural balance mechanisms, movement towards balance was not assured, at least over some time periods. If there are multiple processes at work, observed event sequences will reflect the operation of these processes—and these processes may have different priorities for the actors as well as different time scales.

A detailed examination of the sequences of events generated in the simulations will provide some guidance in the construction of narratives of event sequences as advocated by Abell (1987). While potentially confusing, studying event sequences seems the most useful way of discerning the operation of balance theoretic processes. With that knowledge gained, generalized mechanisms and their operation can be elaborated. We may then be in a position to specify system causality models and estimate them. But understanding event sequences and the operation of algorithmic rules may be the understanding of network evolution that we seek and so limit system causality models as summary devices (that may still be useful for understanding dynamics).

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