

## A BRIEF HISTORY OF BALANCE THROUGH TIME

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We present methods for establishing the amount of reciprocity, transitivity and group balance (a generalization of structural balance) in sociometric structures. These methods are applied to the second time series of sociometric data provided by the Newcomb (1961) study. The amount of reciprocity was above chance levels at the outset and showed no systematic variation thereafter. Transitivity has a very different time scale. It climbed steadily through the first nine weeks and remained stable thereafter. While consistent with chance at the beginning of the study period, it grew to above chance levels at week 3. Group imbalance declined throughout the entire period. The reasons for these different time scales are discussed.

### 1. INTRODUCTION

The term "structure and process" is used so frequently in the social scientific literature that it seems hard to study one without the other. Yet within the study of social networks it seems that "structure" and "process", in the main, have been uncoupled. The strand of literature from White (1963) through Lorrain and White (1971) to many of the blockmodel analyses in the social network literature has focused primarily on "structure". This research tradition has been prominent in network analysis (Hummon and Carley, 1993) and has been very fruitful through being deeply structural. It fits well within the structural paradigm where networks are represented in graph theoretic terms.

However, there is a potential drawback that stems from this focus on structure. Attention can be confined to 'the group structure' represented in graph theoretic terms, and to structural analyses of a particular graph or to a set of closely linked graphs. In reaction to this, Barnes and Harary (1983) echoed Doreian's (1980) con-

cern that structural analyses tended to have a static cast.<sup>1</sup> This may be an over-reaction—or it may be that the times have changed.

More and more, network analysts are bringing temporal concerns into their analyses of social networks. See, for example, the contributions to Weesie and Flap's (1990) edited volume and those in Doreian and Stokman (1996)—another edited collection. There has been a growing recognition that some of the classic data sets do have time referenced data.<sup>2</sup> These include the Sampson (1968) data and Newcomb's (1961) study (Nordlie, 1958).

Time can be included in network analyses in many ways, some of which are more ambitious than others. The most ambitious would have a full dynamic theory that modeled the processes that change and maintain both the micro structure of a network and its macro-structure. Both levels of analysis would be fully integrated with a clear understanding of the time scales of the phenomena being modeled. Actors would be free to enter and leave the network. We are not that ambitious here.

Our focus is on balance processes in human groups. More specifically, we consider two micro-level processes—reciprocity and transitivity—and a macro-level structural characteristic. This is the “group balance”, a generalization of structural balance. We develop some tools for the micro-level processes and explore some recently created tools that are useful for examining the macro-level structure. We apply all of these tools and ideas to some of the data generated by the Newcomb (1961) study which are distributed as a part of UCINET (Borgatti, Everett and Freeman, 1992).

### 1.1. Reciprocity

Since Moreno (1953) developed the sociogram as an analytical tool, social scientists have been interested in the degree of reciprocity among actors in human groups. Reciprocity has been argued theoretically as being essential for the effective running of complex societies. Axelrod and Hamilton (1981) used simulations to demonstrate that reciprocal behavior is a dominant strategy for dealing with problems that require coordination. Socio-biologist Rivers (1983) suggested that tendencies toward reciprocal friendships are genetically ingrained to increase the probability of survival. Gouldner (1960) proposed that there is a universal norm of reciprocity that obligates people to reciprocate each other's pro-social advances.

These tendencies towards reciprocity have been found both in the laboratory and in the field (Backman and Secord, 1959). Since then, experimental results have supported this claim—with the caveat that there is fine line between sincere liking and ingratiation (Mettee and Aronson, 1974). If the sender of a friendship tie is discovered to be insincere or acting friendly with ulterior motives, then the receiver of the sent tie is likely to not reciprocate. (See, for example, Jones and Pittman (1982).) Thus, there are conditions under which reciprocity can be an unstable phenomenon.

<sup>1</sup>Doreian and Stokman (1995) estimate that about 15% of the articles contained in the first 16 volumes of *Social Networks* dealt with time in some fashion. Many—but certainly not all—network data sets are defined for a single point in time. See, for example, Kapferer (1969), Thurman (1989) and Krackhardt (1987).

<sup>2</sup>As network analysts create new network tools they have used the classic data sets as test beds for these structural techniques and, in doing so, discarded the temporal information.

In the field, consistently, reciprocity has been shown to exist (Homans, 1950) and at well above chance levels (Wasserman, 1987, 1980; Holland and Leinhardt, 1981). When reciprocation is not present, it is often the case that those sending ties withdraw their friendship nominations (Goffman, 1963). In a set of school children, Hallinan (1978/1979) found that reciprocated friendships endured twice as long as non-reciprocated friendships.

Hallinan's work constitutes an exception to the general practice of static analyses of networks. Her research rested on an ergodic model of transmissions from one time period to the next, without exploring the differences in patterns of reciprocities as the groups evolved. Our intent is to build on her work by examining how reciprocity might increase, decrease or remain stable as a group matures.

## 1.2. Transitivity and Group Balance

Interest in both transitivity and structural balance stems from Heider's (1946) model of balance in people's attitudes towards objects, both social and physical.<sup>3</sup> Heider recognized that transitivity is another integral component: "[T]he case of three positive relations may be considered psychologically ... transitive" (Heider, 1958: 206). More generally, triads beyond the all positive triad also tend towards a state of balance. Heider (1946) assumes people prefer balanced structures in their cognitive representations of these structures and their affect content. Further, if these structures are not balanced, actors experience "strain" and "tension", both of which generate forces towards balance.

Cartwright and Harary (1956) provided a mathematical formalization of Heider's theory, one that laid the foundation for additional theoretical contributions. Heider's theories were couched in terms towards balance. This implies that, at least for some points in time, the structures are not perfectly balanced and that measures of imbalance area needed. One such measure was provided by Cartwright and Harary via a relative count of balanced and imbalanced triads. Each triad can be viewed as a semicycle and a semicycle is balanced if it contains an even number of negative lines. Equivalently, the sign of the semicycle is the product of the signs it contains and a semicycle is balanced if its sign is positive. The measure of imbalance is a (weighted) ratio of the number of imbalanced semicycles to the total number of semicycles.

As Rapoport (1963: 541) noted, the Cartwright-Harary theorems on balance generate four rules about combinations of friends and enemies among a triad of actors:

1. A friend of a friend will be an friend;
2. A friend of an enemy will be an enemy;
3. An enemy of a friend will be an enemy;
4. An enemy of a enemy will be an friend.

All of these balance models assume that relations among actors are either binary (friend or non-friend) or trinary (friend, neutral or enemy). Freeman (1992) has brought to our attention that one can draw upon mathematical theory to assess the

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<sup>3</sup>He is credited with the first systematic statement of this important area of social psychology.

same degree of transitivity in continuous data. He noted that an ultrametric, representing a measurement of distance  $d_{ij}$  between objects, has the following properties for all  $\{ijk\}$  triples:

1.  $d_{ii} \leq d_{ij}$
2.  $d_{ij} = d_{ji}$  (Symmetry condition)
3.  $d_{ik} \leq \text{Max}[d_{ij}, d_{jk}]$  (Transitivity condition).

The last condition preserves transitivity for any given distance level in the data. That is, at some arbitrary cutoff distance, say  $d_{ij} = C$ , we can dichotomize the data so that

$$M_{ij} = \begin{cases} 1 & \text{if } d_{ij} \leq C \\ 0 & \text{otherwise} \end{cases}.$$

If  $d$  is an ultrametric, then transitivity will hold in  $M$  for any given level of  $C$ .

The detailed examination of the triads focuses on micro-level processes and local properties of the structure. Another avenue of inquiry was opened by Cartwright and Harary's formalization. The key result of the formalized version of Heider's theory is the structure theorem: If a graph is balanced, the nodes can be partitioned into two subsets such that all of the positive ties are within subsets and all of the negative ties are between subsets. This is a fundamental macro-level feature of the total structure represented in the graph. Davis (1967) presents an extension of this result where a semicycle is imbalanced if it contains a single negative tie. If all of the semicycles are balanced, in this (modified) sense, then the (modified) structure can be partitioned into two or more subsets where the positive ties are within subsets and the negative ties are between subsets. Davis refers to the sets as plus-sets, a usage we will continue. In the Davis extension, the fourth rule identified by Rapoport—an enemy of an enemy is a friend—is relaxed. One consequence of using an ultrametric is that it allows the uncovering of partitions into two or more subgroups of actors, a feature we wish to exploit. However, for our purposes, the problem with using an ultrametric is that it excludes asymmetry. We will relax this condition while keeping the rest of the ultrametric properties. This and using the extended version of the structure theorem permits us to measure the extent of both transitivity and group balance in networks.

The data we use come from the Newcomb (1961) study. We note that the ideas of strain and tension, identified as parts of Heider's theory, are present also in Newcomb's version of balance theory. For our purposes, this is important as "objective" structures and subjective perceptions of those structures are both legitimate objects in his theory. While the latter point to intra-actor processes, consistent with Heider, the presence of the objective structure within the overall theoretical framework creates the opportunity to examine its macro structural features in terms of balance processes.

## 2. METHODS

Our focus is on group balance. The data from the Newcomb (1961) study take the form of a series of 15 matrices where each of the 17 actors, in the pseudo-fraternity, ranks the other 16 actors with regard to affect (Nordlie, 1958). These matrices of

ranks are ordered in time: with one exception, they were obtained at weekly intervals. This is less than ideal, particularly having only the rankings. We have chosen to not use the ranks as if they come from a well defined metric. Intuitively, it seems that only some (small) numbers of ranks at the “positive” end need to be considered and, for a balance, a small number of ranks at the other end of the affect dimension. Differences in the assignments in the middle ranks are taken to be random. In short, actors are clear about which other actors they like and those they dislike.

These considerations imply the use of some data transformations as documented below. For reciprocity, we used optimization procedures to determine the number of liking rankings we needed to take seriously. These results also informed our analysis of group balance.

**2.1. A Method of Reciprocity Analysis**

In most prior work, reciprocity has been considered a polychotomous variable—a dyad was reciprocated, asymmetric or null. For us, considering reciprocity is more complicated due to the Newcomb data containing only ranks. It seems unreasonable to assume that differences in ranking at one end of the ‘scale’ (say, between ranks #1 and #2) correspond to the same difference in ranks at some other point of the scale (say, between ranks #9 and #10). Additionally, there is the ambiguity around interpreting the numbers themselves. If student *A* ranks student *B* at #3 (on a liking list) does *A* actually like *B*? To the extent that *A* does like *B*, how much liking does *B* have to return in order for the relation to be considered “reciprocated”? If *B* likes *A* more than *A* likes *B*, is *A*’s liking reciprocated? For that matter, is *B*’s liking reciprocated?

These are troublesome questions for a researcher trying to determine the extent of reciprocity in the Newcomb data. However, this does not mean there is no information in the rankings, nor that reciprocity cannot be assessed in these data (Wasserman, 1980). The question becomes, then, how can we interpret or transform these rank data to best uncover the reciprocity that does exist?<sup>4</sup>

Our departure point is that reciprocity,  $\rho$ , is most closely attained when a particular *scaled* amount of liking is reciprocated. By scaled, we mean that a weighted numeric value  $l_{ij}$  assigned to the liking of *j* by *i* preserves interval scaling properties. Thus

$$\rho_{ij} = |l_{ij} - l_{ji}| < \rho_{kl} = |l_{kl} - l_{lk}|$$

implies that the dyad {*ij*} is more reciprocated than the dyad {*kl*} and that the difference is interval.

The overall degree of reciprocity for any given time *t* is then given by

$$\rho_t = \frac{1}{N(N-1)} \sum_{i \neq j} |l_{ijt} - l_{jit}| \tag{1}$$

where *N* is the number of actors and  $l_{ijt}$  is the weighted assigned liking value given by actor *i* to actor *j* at time *t*. We note that  $\rho_t$  is an inverse measure of reciprocity—larger values indicate less reciprocity. Therefore, we will maximize reciprocity by

<sup>4</sup>Given the prior literature on human groups it is inconceivable that there is no reciprocity.

finding the minimum value of  $\rho$ . Our task, then, is to assign reasonable weighted values of  $l$ , given the rankings, in order to minimize  $\rho$ .

Using the degree of reciprocity as our criterion, our task is to find the weights that maximize the observed reciprocity in the data. Let  $S(x) = y$  denote assigning a weighted value  $y$  to a rank value  $x$ . The rank values,  $x$ , range from 1 to 16 in the data and we (arbitrarily) set the range of assigned weights,  $y$ , as 0 to 15. To maximize the reciprocity ( $R$ ) found in the data is to minimize the absolute discrepancies in the weights  $l_{ij}$  and  $l_{ji}$ . As some arbitrary weightings make it quite easy by chance alone to attain reciprocity, each aggregated reciprocity score,  $R$ , was divided by  $E[R]$ , the expected value for  $R$  for all  $l_{ijt}$  assignments possible for a given set of mappings,  $S$ .

Formally, we

$$\begin{aligned} \text{Maximize: } & \frac{R}{E[R]} \\ \text{Subject to: } & S(1) = 15 \\ & S(16) = 0 \end{aligned}$$

where:

$$R = \frac{\sum_{t=1}^T \sum_{i \neq j} |S(l_{ijt}) - S(l_{jit})|}{N(N-1)T} \quad (2)$$

and  $T$  is the number of time periods ( $= 15$ ).

The expected value  $E[R]$  was based on the assumption that each person,  $i$ , randomly assigns a unique ranking from 1 to 16 with equal probability to each of the 16 others. The probability that any given rank will be reciprocated with any other given rank is also equal. Since these probabilities do not change over time, the expected value for any time point will equal the overall expected reciprocity across all time periods:

$$E[R] = \frac{\sum_{i,j=1}^N |S(i) - S(j)|}{N(N-1)}. \quad (3)$$

The search across the solution space for  $R/E[R]$  was conducted using a Newtonian steepest descent method contained in Microsoft Excel. Initial values for the optimization routine were set to  $S(i) = 16 - i$  for  $i = 2, \dots, 15$ . Thus, the procedure sought to adjust the weights  $S(2), S(3), \dots, S(15)$  with  $S(1)$  and  $S(16)$  anchored at their given values of 15 and 0 respectively.

## 2.2. A Method of Transitivity Analysis

As the data are forced rank, we encountered two problems in using the traditional definition of an ultrametric. We have mentioned already the first: ultrametricity requires both symmetry and transitivity. Since we measure symmetry separately, we simply ignore this requirement. So when we refer to a triple as not violating the property of ultrametricity we mean that it does not violate the *transitive condition* of ultrametricity.

The second problem is more consequential. Rank orders are restricted in their use and this restriction itself can prevent the arrangement of ranks from obeying the transitive condition. To illustrate this, consider the following simple example. Suppose that  $i$  rates  $j$  as #1 and, further, that  $j$  rates  $k$  as #1. The only way to preserve ultrametricity in the  $\{ijk\}$  triple is for  $i$  to rank  $k$  as #1. But, by the rules of forced rank,  $i$  has already nominated  $j$  as #1 and cannot assign that rank to  $k$ . As a result, there is no rank that  $i$  can assign to  $k$  and preserve ultrametricity.

This problem can be alleviated, however, if the transitive condition of ultrametricity is modified to the following:

$$\text{Max}(d_{ij}, d_{jk}) + K \geq d_{ik} \tag{4}$$

$K$  gives more latitude to the ranks to preserve the ultrametric—the larger the value of  $K$  the easier it is to retain transitivity. A value of  $K = 1$  is sufficient to preserve the ultrametric in the above example. It is also minimal in that it does not allow an excessive number of triples to remain transitive as an artefact concerning the choice of  $K$ . So, in the following examination of transitivity,  $K = 1$ .

We started with the following indicator of transitivity  $\tau$  for the triple  $\{ijk\}$  at time  $t$ :

$$\tau_{ijk} = \begin{cases} 1 & \text{if } d_{ikt} \leq \text{Max}(d_{ijt}, d_{jkt}) + 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The degree of transitivity is then defined as the total number of transitive triples—that is, triples that do not violate the transitive condition of ultrametricity—within the data set of ranks.

It should be noted that, unlike the analysis of reciprocity, no optimum weights were calculated for transitivity. Ultrametricity differs from reciprocity in one important way: the indicator of transitivity,  $\tau$ , is a function only of the rank orders in the  $d_{ijt}$  values. Arbitrarily weighted values would play no part in the calculation of  $\tau$  as long as the weighted rankings preserved the original rank order—which they are forced to do. Finding an “optimum” weighting scheme has no relevance, nor would it make sense.

### 2.3. A Method for Group Balance

We consider next balance processes for small groups. As noted above, a semicycle is balanced if the product of all of its ties is positive and a graph is balanced if all of its semicycles are balanced. We base our method on the structure theorem<sup>5</sup> of Cartwright and Harary (1956) and its generalization by Davis (1967). As noted above, the only difference between the two theorems is the number of plus-sets permitted in balanced graphs. We use the term *k-balance* for balance where  $k$  denotes the number of plus-sets. For  $k = 2$ , we have the original meaning of structural balance and for  $k > 2$ , we have generalized (group) balance. Henceforth, we use the term balance generically to cover  $k$ -balance for all values of  $k$ . From the context it is clear which type of  $k$ -balance is considered. When the nodes are individuals

<sup>5</sup>We use the term ‘lines’ to include both arcs and edges in order to consider both directed and undirected graphs.

and the social relations have affect, the structure theorems are statements about the structure of the whole group.

Considering the group as a whole, the balance theory *hypothesis*<sup>6</sup> states that the social structure of the group's network tends towards balance through time. Stated in these terms, it is clear that the structure theorems do not provide much help empirically as they refer to balanced structures. The empirical hypothesis implies the existence of *imbalance* en route towards balance. In this context, we need two things: (1) a practical way of establishing partitions of the nodes to obtain the plus-sets and (2) a way of measuring imbalance.

We consider the latter need first. There are two broad approaches to measuring imbalance in signed graphs (Harary, Norman and Cartwright, 1965). We have mentioned already the approach in terms of semicycles: the (weighted) sum of balanced semicycles is computed as a proportion of the (weighted) sum of all semicycles. The second approach counts the smallest number of lines for which a sign reversal of *all* of them creates a balanced graph while the sign reversal of all strict subsets of these lines does not do so (Harary et al., 1965: 349). We focus on the latter type of measure as it leads directly to an efficient algorithm for establishing partitions into plus-sets.<sup>7</sup>

Doreian and Mrvar (1996) propose a straightforward way of establishing partitions of nodes of a signed network (graph) that are as close to balance as is possible to a partition conforming to exact balance. Such partitions then provide a measure of the amount of imbalance in a signed graph—there are no other partitions, into a specified number of plus-sets, with a lower measure of imbalance. Their method is a local optimization procedure with the same philosophy advocated by Ferligoj et al. (1994) in the context of blockmodeling. Compared to exact balance, regardless of  $k$ , there are only two kinds of departures from balance: (1) positive lines between plus-sets and (2) negative lines within plus sets. The obvious criterion function is a count of the number of these departures. Partitions, for a given value of  $k$ , are then sought to minimize this criterion function.

Formally, to locate 'optimum' partitions we seek partitions that minimize  $I_b$ , the amount of imbalance, with  $I_b = \sum_p + \sum_n$  where  $\sum_p$  is the number of positive links between plus-sets and  $\sum_n$  is the number of negative links within plus-sets. This measure can be defined for both valued and binary graphs and it is possible to weight the two kinds of departures from balance differentially. Here, we weight them equally.

We apply Doreian and Mrvar's procedure to the Newcomb data. As it was for reciprocity and for transitivity, the ranked form of these data is problematic. In order to apply balance theory ideas we need a delineation of positive and negative ties. In the results section (3.1 below) we establish the utility of focusing on the top four nominations made by each actor at each time point. We use that here and take the top four ranking nominations as positive. We use the bottom three nominations

<sup>6</sup>We emphasize the distinction between the structural balance hypothesis as an *empirical* claim and the *formal* statement about balanced structures.

<sup>7</sup>Empirically, most signed graphs are not exactly  $k$ -balanced. Even so, we continue to use the term plus-set in empirical contexts even if there are *some* negative lines between nodes in a cluster (plus-set).



TABLE 1  
Reciprocity Optimal Weights

Rank	Optimal Weight
<b>1</b>	<b>15.0</b>
2	15.0
3	15.0
4	15.0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
<b>16</b>	<b>0</b>

• Bold faced values indicate anchor values that were not optimized.

made be each actor as negative ties, again at each point in time. Nordlie (1958) notes that individuals receiving the bottom three nominations are “unpopular”. For the data collection, each member of the pseudo-fraternity was asked to sort the names of the other 16 members into piles that included a “dislike” pile. There was a bias away from using the negative label: seldom were more than three negative nominations made by an individual.<sup>8</sup> Consistent with Nordlie’s report, we use the bottom three nominations as negative ties. The remaining ranked nominations from 5 to 13 are treated as null.<sup>9</sup>

### 3. RESULTS

#### 3.1. Reciprocity Results

The minimization search procedure described in Section 2.1 yielded the assigned weights shown in Table 1. These weights suggest that reciprocity is maximized under the conditions that the first four nominations (ranks 1, 2, 3 and 4) are given the maximum weight (15) and all of the other ranks are given the minimum weight (0). Without loss of generality, we achieve the best results when the top four choices are a ‘1’ and the remaining choices are ‘0’.

This is a remarkably strong result. It suggests that the degree of reciprocity in the data is best captured by greatly simplifying the ranks to a set of 1’s and 0’s—specifically with four 1’s per individual nominator. This realization makes calculating

<sup>8</sup>Nordlie reports that, frequently, only one or two negative nominations were made, consistent with the bias away from reporting negative ties (even if they exist and are experienced).

<sup>9</sup>In doing so, we assumed that the differences in the relative rankings in the middle of the distribution or ranks were inconsequential while those at the extremes are important. Actors know who their real ‘friends’ and real ‘enemies’ are. The qualitative results (below) hold also when the top three ties are coded positive and the bottom three are coded negative, and when the top three are coded positive and the bottom two are coded negative.

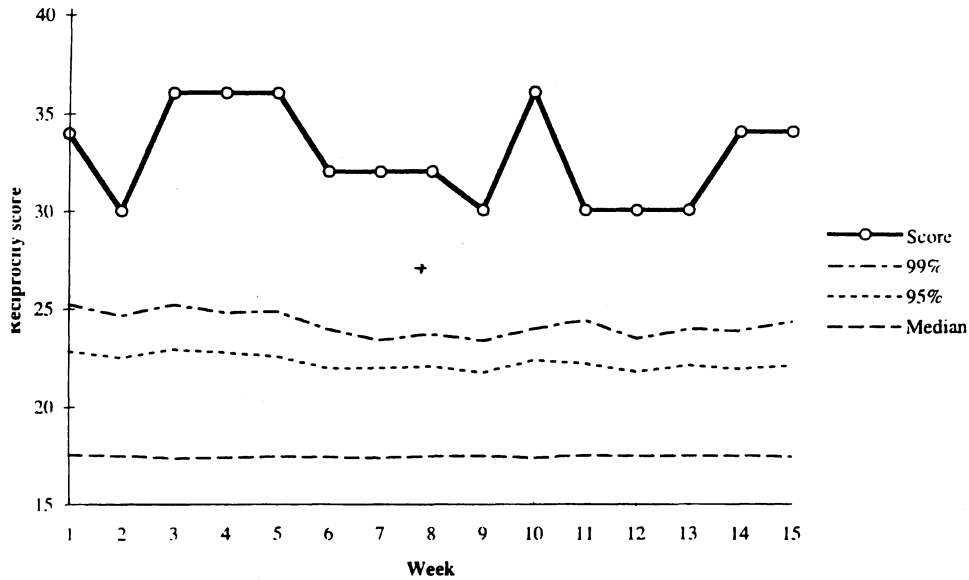


FIGURE 1.

the degree of reciprocity much easier: a sent friendship tie is reciprocated if the target sends a choice back. There is no need to worry about whether the ranks are close enough to warrant calling an arc a reciprocated tie.

Once we recode these ranks appropriately, we can plot the extent of reciprocity in these data across time. See Figure 1. To do so, we calculated a week-by-week index of the degree of reciprocity by simply counting the number of reciprocated ties in the recoded data. This value,  $R_t$ , can take values from 1 (where no ties are reciprocated) to 68 (where all ties are reciprocated).<sup>10</sup> Establishing this range of variation, however, does little to help us determine whether a particular degree of reciprocity is “large” or “small”. In particular, it would be useful to know whether the number of observed reciprocated ties is greater or less than what we would expect by chance arrangement of the ties, given their density and structure. To do this, we draw on the fact that the degree of reciprocity can be calculated as follows:

$$R_t = u' M \cdot M' u \quad (6)$$

where  $u$  is a column vector of 1's and  $M$  is the matrix of 0's and 1's, with 1 representing the presence of a tie from  $i$  to  $j$  at time  $t$ , and  $\cdot$  is an element-wise matrix multiplication. (That is  $C = A \cdot B$  implies that  $c_{ij} = a_{ij}b_{ij}$ .) The result is a count of the number of reciprocated ties.

<sup>10</sup>Having all ties reciprocated, in principle, does rest on the number of ties being sent by each actor ( $T$ ) and the number of actors ( $N$ ) in the network. In this case, it is straightforward to show that the maximum of  $NT$  reciprocated ties can be obtained. To show this conceptually, imagine all  $N$  actors arranged in a circle and require each actor to send  $N/2$  ties to her right and  $N/2$  ties to her left. As long as  $T$  is even, each actor's ties will be reciprocated. Note that for  $T = 3$  and  $N = 6$ , all ties can be reciprocated which implies that having  $T$  even is not a necessary condition.

There are several ways we could calculate the distribution of  $R_t$  under a null hypothesis. We employ the logic of Hubert (1987) to derive the null distribution by generating randomly permuted matrices  $P$  (where  $P$  is permutation of the observed matrix,  $M$ ). We construct a set of  $R_t^*$  values under the null hypothesis as follows:

$$R_t^* = u'P \cdot M'u \quad (7)$$

Each  $R_t^*$ , generated from a randomly generated  $P$ , represents a possible outcome under the null. More importantly, each  $P$  preserves the structure of  $M$  as, by definition,  $P$  is isomorphic to  $M$ . This is a useful feature in generating the possible outcomes under a null hypothesis in network analysis because, by conditioning all of the possible outcomes on the observed structure, the procedure effectively controls for the row, column and reciprocity autocorrelation in the data. As such, inferences made from these null distributions are not statistically biased (Krackhardt, 1988).

We generated 1000  $P$ 's and associated  $R^*$ 's for each time period  $t$ . From this generated null distribution, we calculated the median, the 95th percentile and the 99th percentile values for each  $t$ . By comparing these values with the observed values of  $R_t$ , we can determine whether the observed degree of reciprocity is significantly greater than what would be expected by chance arrangement of the ties. These results are plotted also in Figure 1. The median values under the null hover around 17 reciprocated ties per period. The 95th and 99th percentiles are consistently about 23 and 25 ties respectively. In contrast, the observed reciprocated ties range from 30 to 35 across all periods, well above the .01 significance level indicated by the 99th percentile value.

As can be seen from the plotted values in Figure 1, reciprocity is strong and significantly greater than chance throughout the 15 week period. The results are consistent with what others have found in the Newcomb data: friendships are largely reciprocated immediately. Further, there is little discernible trend either up or down in the rate of reciprocity through time.

### 3.2. Results for Transitivity

Since no optimization was required to analyze transitivity, we calculated  $U$ , the number of triples not violating ultrametricity at each point in time, directly:

$$U = \sum_i \sum_j \sum_k \tau_{ijkt} \quad (8)$$

where  $\tau$  is defined in (5). These values are plotted in Figure 2. As with reciprocity, however, it is necessary to know if these observed values are large relative to what would be observed by chance. We use the same logic as described in Section 3.1 to generate the null distribution. Specifically, we calculated  $\tau$  based on the permuted values of  $d_{ik}$ :

$$\tau_{ijkt}^* = \begin{cases} 1 & \text{if } d_{ikt}^* \leq \text{Max}(d_{ijt}, d_{jkt}) + 1 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where  $d_{ikt}^*$  is the randomly permuted set of values of  $d_{ikt}$ . As for reciprocity, medians, 95th and 99th percentile values were calculated for each time period  $t$ . These are plotted also in Figure 2.

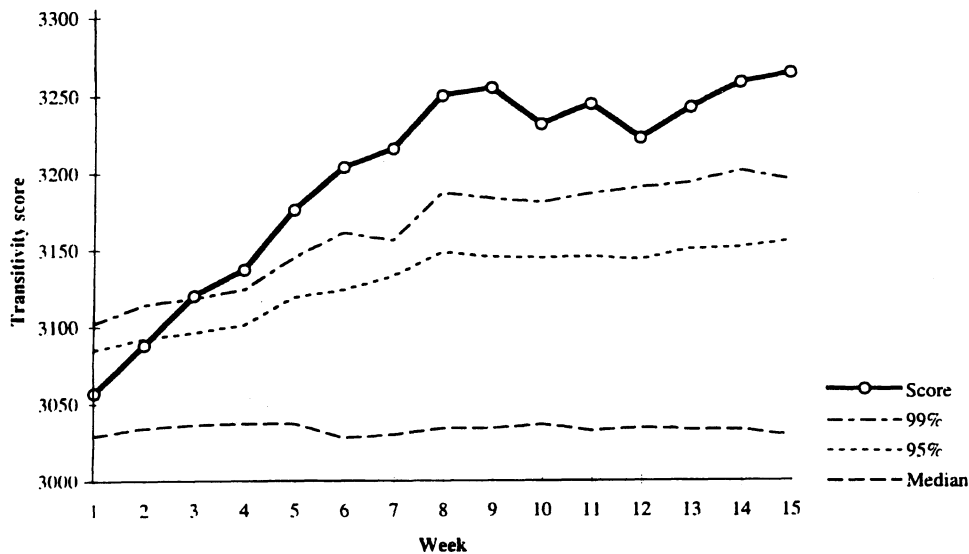


FIGURE 2.

These plots suggest a very different pattern of the emergence of transitive relations over the 15 weeks than was observed for reciprocity. Note the steady increase in transitivity over the first 9 weeks followed by a leveling off through the remaining weeks. Additionally, the amount of transitivity does not reach significance ( $\alpha = .05$ ) through week 2 and barely reaches the .01 significance level in week 3. Thereafter, transitivity is clearly significantly greater than would be expected under chance arrangement of the ties. It appears that transitivity, in contrast to reciprocity, is not immediately reached in the social structure and that it takes much longer to reach its maximum.

### 3.3. Results for Group Balance

The Newcomb data were collected at weekly intervals for 15 weeks, allowing us to consider 14 transitions.<sup>11</sup> Ties between individuals did not remain static over the study period. As described in Section 2.3, we coded the ties as 1, 0 and -1 for the positive, neutral and negative ties respectively. For the  $k$ -balance analyses, if a tie remains positive or negative across a transition, it is stable. Similarly, a null tie that remains null is stable also. Any movement of a tie between pairs of the three states is a change in the dyadic relation. As the people living in the pseudo-fraternity were strangers at the outset, it is reasonable to anticipate more changes at the beginning of the study period compared to later. The top left panel of Figure 3 shows the percentage of dyadic ties that changed.<sup>12</sup> The most change is for the first transition (slightly above 30%). With some oscillation, the percent of ties changing drops over the next 7 weeks and holds steady thereafter with, perhaps a

<sup>11</sup>At one point (only) there is a two week interval between an adjacent pair of observations. We ignored this.

<sup>12</sup>This includes transitions into and out of the null state for a pair of actors.

mild increase over the remaining transitions. Superimposed on the observed data values is a smoothed<sup>13</sup> trajectory that shows these changes more clearly. The top right panel of Figure 3 displays the percentage of dyadic ties remaining stable for each transition as smoothed trajectories. For all ties—positive, null and negative—the percentage not changing at a transition increases over the first 8 transitions and remains stable thereafter.<sup>14</sup>

The graph in the lower left panel of Figure 3 shows the observed trajectories for the imbalance measure  $I_b = \sum_p + \sum_n$  with  $\sum_p$  and  $\sum_n$  as defined in Section 2.3. The upper trajectory shows  $I_b$  for  $k = 2$  (structural balance) while the lower trajectory is for  $k > 2$  (generalized balance). At each time point, the value of  $I_b$  is lowest for the generalized measure of imbalance. Doreian and Mrvar (1996) argue that the lowest value of the imbalance measure, that is, closer to balance, is preferable. Of course, for  $k = 2$ , the focus is on structural balance and other values of  $k$  are ignored. However, for  $k > 2$ , partitions into more than two clusters are considered. For each value of  $k$ , the imbalance measure is computed. Implicitly, many trajectories are examined and compared. Selecting  $k$  for the smallest value of imbalance is more straight forward and less arbitrary than for any other comparison. The substantive balance hypothesis claims a tendency towards balance through time. This is clearly supported for  $k > 2$  (generalized balance). The support is less clear for  $k = 2$  (structural balance) and for both trajectories the change is not strictly monotonic. The lower right panel of Figure 3 displays the smoothed trajectories and the support for the balance hypothesis is unequivocal for generalized balance.

It is worth dwelling on the difference between structural and generalized balance. Tables 2 and 3 display the partitioned structure for the first and last time periods of the Newcomb study. The partitioned structures are very different yet the evolution from the first partitioned structure to that last is very coherent. Initially, there were three plus-sets as shown in Table 2. There are 5 negative ties within plus-sets ( $\sum_n = 5$ ) and 15 positive ties between plus-sets ( $\sum_p = 15$ ). At the second time point (not shown) the plus-set containing A merges with the one containing E. By the seventh time point, the basic structure is close to its final form and gradually stabilizes. Gradually, over the last 6 periods, actors C, J, O and P get detached. For structural balance ( $k = 2$ ), {C, J, O, P} form a plus-set with the other actors forming the second plus-set. The actors in the smaller subset draw more than 80% of the negative ties or, in terms of the ranked data, receive the vast majority of the lowest ranks. The partition in Table 3 shows the partition under generalized balance. The low ranked actors appear as singletons plus a dyad {O, P}.<sup>15</sup> The actors in the small plus-set under structural balance also have negative ties among themselves. The negative ties contribute to  $\sum_n$  for structural balance but they do not contribute to  $\sum_n$  for generalized balance. Hence the lower values of  $I_b$  for generalized imbalance.

<sup>13</sup>This smoothing was obtained using Cleveland's (1979) LOWESS procedure as implemented in STATA (Stata Corporation, 1993).

<sup>14</sup>Of course, for every transition, there are ties that change.

<sup>15</sup>Occasionally, C and O form the dyad with P and J the singletons.

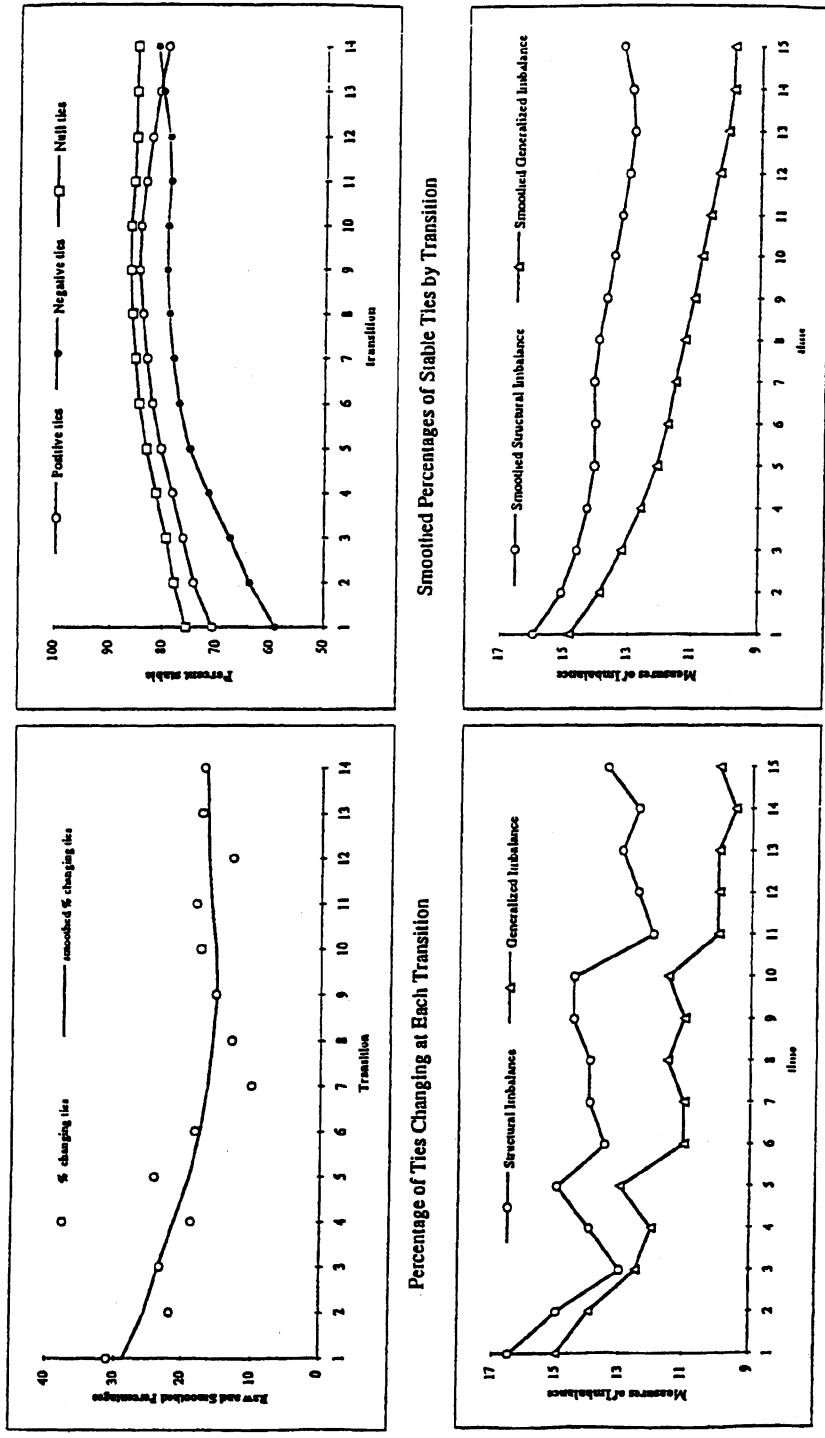


FIGURE 3. Ties changing and group imbalance through time.

TABLE 2  
Partition Into 3 Clusters at First Time Point [Error Score: 15]

	A	F	H	M	B	D	G	N	P	C	E	I	J	K	L	O	Q
A	.	1	-1	1	.	.	.	.	.	.	.	-1	-1	1	.	.	1
F	.	.	1	1	.	1	.	.	.	.	-1	1	-1	-1	.	.	.
H	.	1	.	1	.	.	-1	-1	.	-1	.	.	1	1	.	.	.
M	1	1	-1	.	-1	.	.	.	.	-1	1	.	.	.	.	.	1
B	.	.	-1	.	.	1	1	.	1	-1	.	.	.	-1	.	.	1
D	.	1	-1	.	1	.	1	.	.	-1	-1	.	.	.	.	.	1
G	-1	.	.	-1	1	1	.	.	.	.	-1	.	.	.	.	1	1
N	-1	.	-1	-1	.	.	1	.	.	.	.	1	1	.	.	1	.
P	.	-1	.	.	.	1	-1	.	.	-1	.	1	.	1	.	.	1
C	.	.	-1	-1	.	.	.	.	-1	.	.	.	.	1	1	1	1
E	-1	-1	1	.	.	.	.	-1	.	.	.	.	.	1	1	.	1
I	.	.	-1	.	-1	-1	1	.	.	.	.	.	.	1	1	.	1
J	1	1	.	.	-1	-1	1	.	.	.	.	.	.	-1	.	1	.
K	.	-1	-1	.	.	.	.	-1	.	1	.	1	.	.	1	.	1
L	-1	-1	.	-1	.	.	.	.	.	1	.	.	1	1	.	.	1
O	-1	.	.	.	.	.	.	-1	-1	1	1	.	1	1	.	.	.
Q	.	.	.	.	-1	1	.	-1	.	.	1	1	.	.	1	-1	.

TABLE 3  
Partition Into 4 Clusters at Final Time Point [Error Score: 10]

	A	B	D	E	F	G	H	I	K	L	M	N	Q	C	J	O	P
A	.	.	.	.	.	.	1	.	.	.	1	1	1	-1	-1	.	-1
B	.	.	1	1	.	.	.	.	.	1	.	.	1	.	-1	-1	-1
D	.	1	.	1	1	.	.	.	.	.	.	-1	1	-1	-1	.	.
E	.	1	1	.	.	.	.	1	.	.	1	.	.	.	-1	-1	-1
F	.	.	1	.	.	.	1	1	.	.	1	.	.	-1	-1	.	-1
G	.	1	.	-1	.	.	.	.	1	1	.	.	1	.	-1	.	-1
H	1	.	1	.	1	.	.	.	.	.	1	.	.	-1	-1	.	-1
I	.	.	1	.	1	.	.	.	.	.	.	1	1	.	-1	-1	-1
K	.	1	1	.	.	.	-1	.	.	1	.	.	1	.	-1	.	-1
L	.	1	.	.	-1	1	1	.	.	.	.	.	1	.	-1	-1	.
M	1	.	.	.	.	1	1	.	.	.	.	.	1	-1	-1	.	-1
N	1	.	.	-1	1	.	.	1	1	.	.	.	.	-1	-1	.	.
Q	1	1	1	.	.	.	.	1	.	.	.	.	.	-1	-1	.	-1
C	.	.	.	.	1	.	.	1	.	1	.	-1	1	.	-1	.	-1
J	1	.	.	.	.	1	1	.	.	1	-1	.	.	-1	.	.	-1
O	.	.	1	-1	.	.	.	1	-1	.	.	.	1	.	-1	.	1
P	.	.	1	.	.	.	-1	1	.	1	.	.	1	-1	-1	.	.

4. DISCUSSION

Overall, the results suggest that the emergence of reciprocity in a newly formed group differs substantially from transitivity and group balance. Consistent with Newcomb's (1961) account, we found that reciprocity is established early in the process.

While Newcomb argued that it was established in week 3, our analysis indicates that even during the first week the degree of reciprocity is near its maximum. This suggests that reciprocity is normative. In looking at the number of mutual, asymmetric and null ties, Wasserman (1980: 288) also noted "... [t]he data are remarkably constant, which may indicate the group has reached an equilibrium" (at least for reciprocity). Further Wasserman found that reciprocity increased the likelihood of an  $\{ij\}$  tie in the following period by as much as six times. He concludes that "reciprocated arcs are even more important than Newcomb noted". We concur.

We note also that Newcomb (1979) reports "the nonconfirmation of a plausible hypothesis" concerning reciprocity. He reports "... there were virtually no differences between early and late frequencies of interdyadic discrepancies" (1979: 300). His conclusion rests on a comparison between an early and a late week. We add that the levels of reciprocity are about the same (and high) throughout the study period. Note also, that the partitioned structures in Tables 2 and 3 make it very clear that there are limitations on the amount of reciprocity in this small group. The four actors that were identified as receiving over 80% of the negative ties cannot have reciprocated positive nominations. These actors were also identified quite early.

Contrary to this early stability in reciprocity, however, the results for both micro-based transitivity and macro-based group balance indicate that an equilibrium was not reached until much later. The rate of changes in ties levels off around week 9, about the same time that transitivity stops increasing. While more erratic, group balance appears to increase almost to the end of the 15 week period.

One explanation for these results is that, while people are re-evaluating their friendship choices as time goes on, reciprocity acts almost immediately on those choices. Reciprocity is both a simple and a strong mechanism influencing such choices. As we suggested earlier, the norm of reciprocity is universal and frequently difficult to break (Gouldner, 1960). To choose reciprocally, each actor  $i$  needs only to understand the  $i \rightarrow j$  tie. Perhaps, even more importantly, reciprocated ties are primarily a local phenomenon with relatively little impact on the rest of the structure.

Transitivity, on the other hand, is more complex. First, for an individual to choose to make a transitive tie from  $i$  to  $k$ , she must be aware of at least two other ties,  $i \rightarrow j$  and  $j \rightarrow k$ , one of which she is not involved in directly. Second, the addition or deletion of an  $i \rightarrow k$  can have implications for the degree of transitivity for many other actors in the network. For example, the addition of  $i \rightarrow k$  to complete transitivity for  $i$  upsets the transitivity of all actors,  $q_1, q_2 \dots$  who are tied to  $i$  but not to  $k$ . As opposed to reciprocity, the "filling in the blanks" to make relations transitive has a ripple effect throughout the structure.

While reciprocity and transitivity dynamics are local phenomena (where the choices of individuals can be viewed as constrained by balance tendencies) group balance is a macro-phenomenon. It seems unreasonable to think in terms of single actors making choices that are constrained directly by group balance. However, the micro-level processes can be viewed as generating social forces that move the structure towards group balance. The time scale of these macro-level dynamics will be longer than the relatively simple reciprocity and transitivity phenomena. It is quite possible that two or three people change relations, when they are involved, from



neutral to positive (or negative) ties and create a new “optimal” structure that is different to the composition before the switch. Thus, at this most global, level it is not surprising that group imbalance, as shown in Figure 3, continues to decline for a longer period.

We have noted that some of the results reported here are consistent with prior analyses. However, they also diverge from other accounts of the changes through time that were experienced by this group of actors. Nakao and Romney (1993), while examining different structural characteristics, report that their results are “basically consistent” with the conclusion arrived at by Nordlie (1958) and Berger et al. (1995): the overall attraction pattern stabilizes in the third week to fifth week. We suggest that the time towards stability differ according to the structural properties that are considered. Some phenomena are more complex than others and take longer periods of time to reach an equilibrium distribution (if one is reached).

Different groups may have different dynamics and it will be useful to explore these issues across different groups. For example, the partitioned structure of this human group differs greatly from the partitioned structure of the Sampson (1968) data re-analyzed by Doreian and Mrvar (1996) using the same balance techniques. In the monastery there were three clearly identified subgroups that were plus-sets internally and were mutually antagonistic. For the students in Newcomb’s pseudo-fraternity there were, in principle, opportunities for interactions outside the fraternity. While some individuals receive many negative (or very low) evaluated ties within their residence, they may have their positive relations elsewhere. The trainee monks were isolated from the outside world in terms of interaction opportunities. It seems a reasonable hypothesis that one of the features of total institutions is a tendency to generate internally strong but mutually hostile subgroups or a single cohesive subgroup to which all actors belong.

It may also be the case that there are some properties that are consistent across many groups. Doreian and Mrvar (1996) noted that, in the Sampson data, there were many more positive ties between plus sets than negative ties within plus-sets. They speculated that this may be a general property of human groups: internal subgroup negative ties are much less tolerated than positive ties between groups. The former are threatening to the subgroup while the latter could be tolerated idiosyncratic nominations. Interestingly, this pattern characterizes Newcomb’s pseudo-fraternity also.

Finally, we have one comment on the content and form of the Newcomb data and the kinds of analyses that can be undertaken with them. Although we opted not to use the ranks reported by Nordlie (1958), we recognize that many useful analyses have been undertaken with these measures. Nakao and Romney (1995) continued that tradition. Although we focus on different structural features, use different tools and reach different conclusions concerning the time scales for the emergence of social structural features, we fully concur when they write “... longitudinal information provides us with new knowledge and an additional perspective in explaining some structural phenomena in small group situations that we would never be able to obtain from cross-sectional research” (1993: 109). This is one of the rich legacies of the Newcomb study—longitudinal data on sociometric structure. At some future point it will be fruitful if the different analyses can be synthesized into

a single coherent theoretical statement. We hope we have taken a step in this direction.

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