



## Reflections on Studying Signed Networks

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### Abstract

Despite considerable success, the balance theoretic approach to studying signed relations has encountered some serious problems, both substantive and methodological. The more consequential problems are outlined along with arguments for why solving them is so critical. An agenda of research problems is laid out with many juicy problems to solve. These reflections, while setting a context in prior work, are far more concerned about looking to the future and identifying problems whose solutions hold the potential for transforming the field.

### Keywords

Signed networks; Structural balance; Relaxed structural balance; Multiple processes; Open problems.

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I was surprised by, and most appreciative of, the invitation to reflect on studying signed networks for this special issue. While there are multiple approaches to examining such networks, the tack taken here deals with one line of work with which I am most familiar. These reflections have two broad components. One is to provide a partial summary of this fruitful line of research. More importantly, the second component outlines some difficult problems that have emerged for which solutions must be found. While it was tempting to confine attention to some of the successes of this approach, these serious problems raise major questions. Further, if these problems are solved, other avenues of inquiry will be opened. Future challenges are more consequential than past successes. So, this reflection can be read also as an invitation to join the quest.

## I STRUCTURAL BALANCE THEORY

For too long, social network researchers confined their attention largely to networks featuring only positive ties. Yet, human relations have been known for a long time to be signed. An

important early paper on signed relations is the statement by Heider (1946) which was foundational for the line of research considered here (Taylor, 1970). As with most profound statements, it was a simple one. Its essential elements are shown in Figure 1. Also, it was substantively based, a critical feature. Indeed, there is far more to doing social network research than the analysis of network data (Robins, 2015). From a social science perspective, there has been an unfortunate separation of substance and methods within some social sciences. A substantive account of social phenomena is a logically consistent account attempting to explain the phenomena using theoretical ideas. Methods can be viewed as a set of techniques designed to analyze data to test theories. When theories are developed with little or no regard for data or when methods are developed as ends in and of themselves, this disjunction is problematic. Ideally, the two need to be integrated in a coherent fashion. See Brusco *et al.* (2011) for a fuller account of this distinction.

Heider's idea came from thinking about the cognitive images three actors, denoted by  $p$ ,  $o$  and  $q$ , could have of their signed relations. Positive ties are represented by solid lines in Figure 1. The dashed ties represent negative ties. According to Heider, the triples in the top row are 'comfortable' for  $p$ ,  $o$  and  $q$ . Reading across the top row of Figure 1, the triples have been captured by four folk statements: i) a friend of a friend is a friend; ii) an enemy of a friend is an enemy; iii) a friend of an enemy is an enemy; and iv) an enemy of an enemy is a friend. Put differently, they are balanced. In contrast, all the triples in the bottom row were considered as discomfoting. Actors in these triples were thought to be motivated to reduce this discomfort by changing one sign in the triple. Another way of representing the signs is numerical by using 1 (for positive ties) and  $-1$  (for negative ties). The sign of a triple is the product of the signs in it. When the product is 1, the triple is balanced. It is imbalanced if the product is  $-1$ . Figure 1 is organized to separate the balanced and imbalanced triples into two panels.

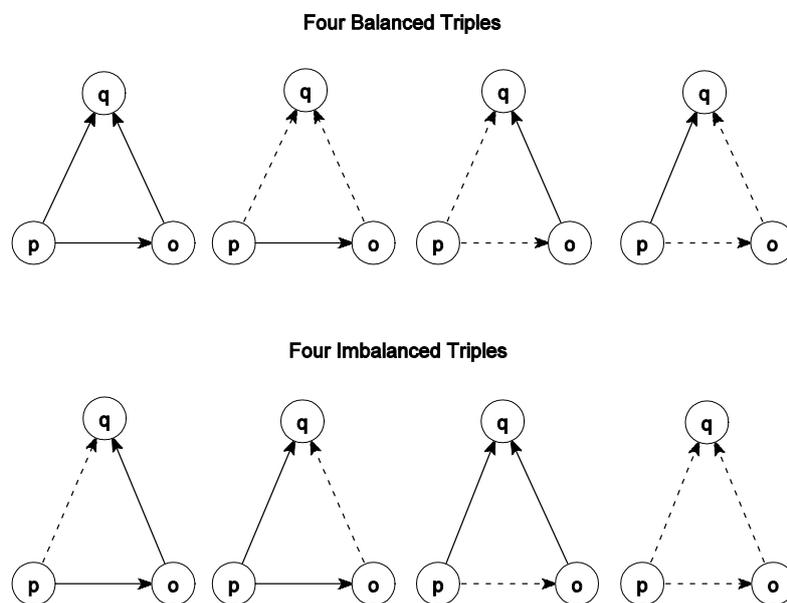


Figure 1: Heider's balanced and imbalanced triples.

Note: Solid lines represent positive ties and dashed lines represent negative ties.

Considering triples in this fashion can be extended to networks by computing the signs of all of the triples in a network. A balanced network has only balanced triples while any network with at least one imbalanced triple is imbalanced. The larger the number of imbalanced triples, the greater the imbalance of the signed network. Two strands of thoughts followed this extension.

The first was methodological, in which the task was to compute a measure for the (extent of) imbalance of a network. An early measure of the imbalance of a signed network is the proportion of imbalanced signed triples among all signed triples (Hummon & Fararo, 1995). Computing this for complete signed networks is unproblematic. But when these networks are not complete, the measure of imbalance can be recast as the proportion of balanced cycles of any length. Computing such a measure turned out to be a difficult computational problem. The second strand was substantive. It took the form a seemingly plausible empirical proposition: Signed networks move towards a balanced state.

Cartwright & Harary (1956) picked up on this idea for signed networks, albeit for networks outside the minds of actors. They formulated and proved a theorem: When a complete signed network is balanced, the vertices representing actors can be partitioned into two subsets of actors with a striking property: All the ties between actors within a subset are positive while all of the negative ties are between actors in different subsets. Davis (1967) noted some human groups split into more than two mutually hostile subsets and suggested the all negative triple (bottom right in Figure 1) be considered as balanced. Doing this led to another theorem where there could be more than two subsets of actors with the same property. Doreian & Mrvar (1996) labeled them ‘the structure theorems’ of balance theory. The results were extended easily to signed networks that were not complete. Letting  $k$  denote the number of clusters (called positions),  $k = 2$  is for the Cartwright and Harary theorem and  $k > 2$  applies for the Davis theorem.

A completely different strand of research for studying social networks takes the form of blockmodeling, where the goal is to take a large (and potentially complex) network and partition the vertices and the ties to create a simpler and far smaller ‘image’ (blockmodel) of the network. The early uses of blockmodeling were instrumental in developing a unique approach to discerning the underlying structure of networks with a concern for structural roles. Most were based on structural equivalence (Lorrain & White, 1971) with two algorithms coming to dominate the empirical analyses. They were due to Breiger *et al.* (1975) and Burt (1976). Batagelj *et al.* (1992a, b) and Doreian *et al.* (2005) provided an alternative conception of how to partition social network while remaining faithful to the blockmodeling enterprise.

Doreian & Mrvar (1996) noticed that the balance structure theorems permitted a clear connection between studying signed networks and blockmodeling. They defined a positive block as having only positive (and null) ties while a negative block has only negative (and null) ties. The structure theorems imply that positive blocks are located only on the main diagonal of the blockmodel and negative blocks are all off the diagonal. They proposed a fast heuristic algorithm for partitioning signed networks based on structural balance. In principle, this can be applied to any signed network. However, partitioning large networks is problematic given this being an NP-hard problem.

The Doreian & Mrvar (1996) procedure used an alternative measure of imbalance taking the form of the number of ties whose sign change (or removal) created a balanced network. This is the line index introduced by Harary *et al.* (1965). More formally, the criterion function for a binary network (where the ties are +1, 0 or -1) is the total number of positive inconsistencies (positive ties in what is thought to be a negative block), denoted by  $P$  and the total number of negative inconsistencies (negative ties in positive blocks) denoted by  $N$ . A general measure of how poorly a blockmodel fits the data is given by a criterion function,  $C_f$ , where  $C_f = \alpha N + (1 - \alpha)P$ , where  $0 < \alpha < 1$ . With  $\alpha = 0.5$ , the two types of inconsistencies are weighted equally, a convention that has been followed consistently. The algorithm was implemented in Pajek (Batagelj & Mrvar, 1998).

Everything looked good both for partitioning signed networks and computing the balance of signed networks. Indeed, many useful analyses were done with substantive implications. But

empirical reality was not so kind for these endeavors because substantive and methodological problems were exposed. The substantive ideas were not supported by empirical data and unproductive ideas were pursued despite these problems. Also, the methods used were shown to have major problems. An examination of them follows in the next section.

## II MAJOR PROBLEMS WITH BALANCE THEORY

The problems start with the basic empirical hypothesis that informed the approach for a long time. Is there really a universal tendency towards balance? The evidence, what little there is, suggests not. Doreian & Krackhardt (2001) examined the well-known temporal Newcomb (1961) social network data (documented by Nordlie, 1958). If the simple empirical hypothesis regarding a universal movement towards balance was correct, then the number of all of the balanced triples of Figure 1 would increase through time while all of the negative triples would decrease. This idea was shattered. Overall, two of the balanced triples became less frequent over time while two of the imbalanced triples became more frequent! While four triples did behave as expected under the empirical hypothesis, the behavior of the other four was bad news.

Hummon & Doreian (2003) conducted a simulation of ‘Heider actors’ behaving in ways fully consistent with Heider’s ideas. Their results were even more devastating. First, many networks did *not* end in a balanced state. The reason for this was quite simple. Attempts to create balance in one triple could – and frequently did – have impacts in other triples. Creating balance in one triple can imply imbalance in one or more other triples. Achieving balance in signed networks is not simple. Second, the sequences of network structures, as the ties evolved over time, were very long with both increases and decreases of imbalance. The implications were obvious: changes in the overall balance of signed networks are not unidirectional. Most certainly, they are *not always towards* balanced configurations. This was an important, albeit largely overlooked, result. See also Robins & Kashima (2008). The basic hypothesis of networks moving towards balance is false. The almost exclusive focus on this hypothesis had unfortunate consequences as described in Section 3.2.

In the context of human signed relations, Doreian & Mrvar (2009) asked another question: Is the blockmodel structure implied by structural balance overly restrictive and counter-productive? They answered in the affirmative. The statement of the structure theorems (described above) implied a specific block-structure with positive blocks on the main diagonal and negative blocks of the diagonal of the blockmodel. The assumption was that structural balance was the only process operating. This changes if other processes are in play. Differential popularity is clearly present in human groups, as is differential unpopularity. The former clearly points to positive blocks *off* the main diagonal of the blockmodel. If they exist and, perhaps, also negative blocks on it, how could signed blockmodeling accommodate this? Asking this question led them to propose the notion of ‘relaxed structural balance’ under which positive and negative blocks could appear anywhere in the blockmodel image. The criterion function for imbalance was not changed. Note this algorithmic change was driven by substantive concerns. For the human networks, they studied, much better fits to the data were achieved. See Doreian & Mrvar (2014) for a more extended study of signed networks using relaxed structural balance. There were positive off-diagonal blocks. And there was a ‘den of vipers’ creating a negative block on the main diagonal of the blockmodel image. If the structural features identifiable using relaxed structural balance are present in a signed network, this helps explain why classical balance theory failed so often to account for the structural changes of signed networks over time.

Signed relations can occur in many contexts. The core idea in structural balance can be reformulated by using a more general notion of *consistency* among a set of ties within a signed network. Of course, the notion of consistency must be specified in ways that will depend on

contexts. As an example, consider the phenomenon of the US Supreme Court overturning earlier decisions. The positive ties are later decisions citing earlier decisions as legitimate precedents. Negative ties are for later decisions overturning earlier decisions completely. Figure 2 shows two triples of decisions. In the top triple Decision 3 overturned Decision 2 which had affirmed decision 1. Yet Decision 3 also affirmed Decision 1. In the second triple, Decision 6 affirmed Decision 5 which also affirmed Decision 4. Yet Decision 5 overturned Decision 4. Clearly, the social psychological presumptions of balance theory do not apply in this context. But the triples seem fundamentally inconsistent in the sense of being contradictory. In contrast, if the two triples each had only positive ties, there would be consistency for the decisions.

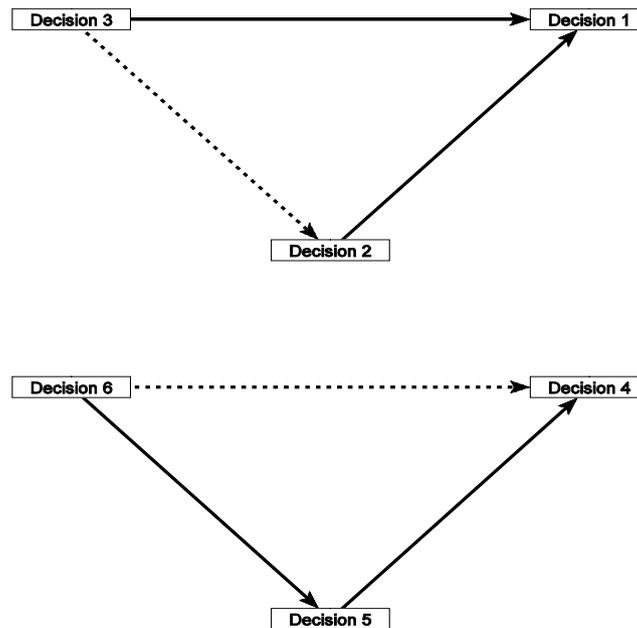


Figure 2: Examples of inconsistent triples for the US Supreme Court.  
Note: Solid lines are affirmations and dashed lines are overturning pairs.

Thinking in terms of signed consistency permits a natural extension beyond interpersonal signed networks. In this spirit, Doreian & Mrvar (2016) studied the structure of the networks of international relations over time using the Correlates of War (CoW) data. (See Pevehouse *et al.* (2004) for details regarding these data.) The definitions of the signed relations they studied are simple to state. First, positive ties between nations are defined by joint memberships in alliances, being in unions of states and sharing inter-governmental agreements. Second, negative ties are for two states being at war, being involved in border disputes, conflicting with each other without military involvement, or having sharp ideological or policy disagreements. This creates a signed network for nations as the actors. Their study featured blockmodeling and measuring imbalance in the overall network<sup>1</sup>.

On tracking balance over time in the international system, Doreian & Mrvar (2016) showed there was no consistent movement towards balance over time. This is revealed clearly in Figure 3 where the amount of imbalance is plotted over time. The underlying assumption was that there would be signed consistency in signed relations but without appealing to social psychological principles. The ideas underlying structural balance were extended to other

<sup>1</sup> Also, when there is a negative tie between states otherwise having a positive tie, the negative tie is used.

networks as an organizing framework in terms of consistency. We were not alone in formulating this extension. See also Mendonça *et al.* (2015) and Vinogradova & Galam (2014). Such approaches can be developed in different fields.

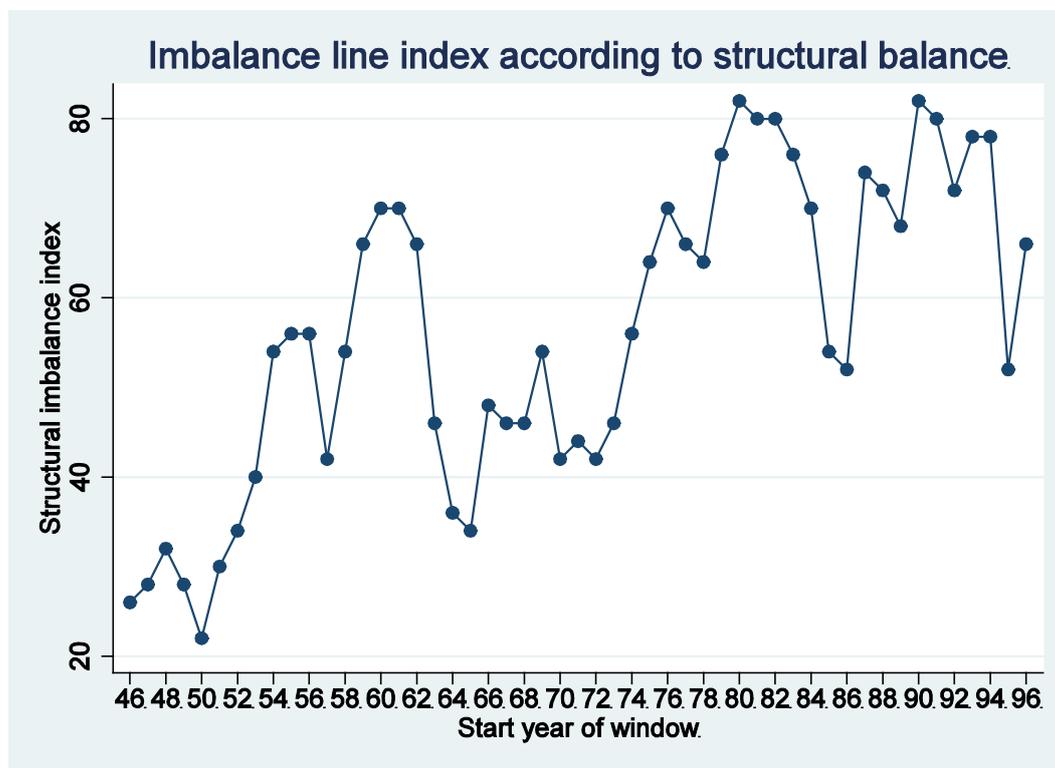


Figure 3: Imbalance measure over time in the international system, 1945-1999.

Establishing the block structures of the signed international relations revealed two serious problems. The first was that the Doreian-Mrvar balance partitioning algorithm failed badly in the way it handled the positive ties. The definition of positive blocks was the problem leading to larger clusters than was reasonable. The second was the failure of the Traag & Bruggeman (2009) community detection algorithm in handling negative ties. Here the problem was the use of modularity which works well for positive ties. But it failed for negative ties by creating a far too fragmented block structure.

Two-mode networks also can be signed. One such network takes the form of Justices in the US Supreme Court (forming one set of objects) agreeing/concurring with or dissenting from decisions made by this court, (the second set of objects). Another example has the voting behavior of nations casting votes in the United Nations General Assembly (UNGA) for or against resolutions. Both involve another construction of what constitutes a signed tie as positive votes for items and negative votes against the same items. Mrvar & Doreian (2009) established an algorithm for partitioning signed two-mode networks, also included in Pajek. An example of a partitioned Supreme Court signed network is shown in Figure 4. The black squares represent votes for decisions, the red squares are for dissenting (negative) votes and white squares are for when justices took no part in a decision. The Justices are partitioned into the assumed conservative and liberal wings of this court for its 1995 term. The decisions, labeled by the substance of the issue involved, are partitioned per the distinct *patterns* of how the justices voted. However, this partition was *not* established with the two-mode partitioning algorithm for it failed completely, especially for clustering the decisions. Indeed, a close look at the pattern of votes suggest it was doomed from the start. The empirical partition structure



will be little chance for the resolution of these conflicts even if such entrenched conflicts are damaging for collective outcomes absent conscious actions to resolve them. It would leave no room for mediation. Doreian & Mrvar (2014) suggested some actors could be motivated to mediate disputes to resolve them. But this also is no more than a speculation. Empirical information would be required to assess whether it is present in mediated conflicts. It could be that such mediators have greater tolerance for imbalance. Most likely, multiple balance processes, far more complex than envisioned by balance theorists, plus the operation of differential popularity and unpopularity, mediation and other mechanisms are likely to be in play in social settings. Understanding the operation of *multiple processes in conjunction* with each other necessarily requires far more complicated theories about the operation of the mechanisms involved in generating signed networks. It will mean also examining and testing *rival* hypotheses. An example of doing so is found in Doreian & Mrvar (2014) in which classical structural balance did not fare well. Alas, the data they used were limited. Much better data are needed for future studies.

Even with a focus on structural properties of networks, where attention is restricted to the network ties, there is a prior issue to consider. Kalish & Robins (2006) studied the psychological predispositions of actors in forming network ties. See also Breiger and Ennis (1979). This goes well beyond the idea of tolerance for imbalance - but is of the same ilk. The formation of network ties depends on: the extant distribution of ties; actor attributes and the *combination* of attributes for *pairs* of actors. It depends also on prior network events involving those actors (Doreian, 2008). This requires far more sophisticated theories about the generative processes, or mechanisms, of signed networks. It is another urgent but difficult problem.

### **3.2 On the failure of the basic empirical hypothesis of structural balance**

As noted above, the basic hypothesis that human signed networks move towards a balanced state is not supported. At best, the evidence is mixed - but most contradicts it. There have been, and continue to be, attempts to assess the 'balance hypothesis' using only positive network ties. All such efforts *must* be dismissed because the dynamics of signed and unsigned networks differ greatly. Such attempts have persisted, especially within US sociology. But this extends more generally. The dynamics of signed relations involve both positive and negative ties: They are equally important. Doreian *et al.* (2015, p.312-3) showed how the separate analyses of positive and negative ties for a social group was useless. Even allowing negative ties in a secondary role to an examination of positive ties (see Esmailian *et al.* 2014) is problematic when studying *signed* relations, a view shared by Mendonça *et al.* (2015).

Even worse, the acceptance of this balance hypothesis as real, with narrowly focused subsequent attempts designed to support it, meant a far more important question was not asked. It is simple to state: *What are the conditions under which signed networks move towards balance and what are the conditions when they move away from it?* As it has been ignored for far too long, the time to address it seriously has come. The trajectories of the signed triples presented by Doreian & Krackhardt (2001) provided one prod to doing this. They suggested that balance *might* work when the tie from *p* to *o* was positive but *not* when it was negative. But this line of thought focused only on the network ties. The movement of the imbalance measure over time shown in Figure 3 also demands that this question be addressed. Of course, signed ties in international relations are very different from signed ties in human social groups. No doubt, the mechanisms involved will differ. But the need to focus on the *contexts* of relations is common to both. A special issue of *Social Networks* (volume 34, issue 1, 2012), edited by Adams, Faust and Lovasi, was devoted to 'capturing context' (Adams *et al.* 2012). It explored contexts for social networks from a variety of vantage points. Empirical settings are far more than the places where the data happened to be collected. This also becomes part of tackling a much deeper substantive problem.

### 3.3 Using substance to inform blockmodeling

The practices in the early uses of blockmodeling, while important for the social networks field, had an unfortunate consequence. Algorithms were switched on, partitions were delineated, these partitions were accepted and then interpreted. This approach to data analysis is a *remarkable expression of ignorance*. Most often, network analysts know far more about the networks they study either substantively or empirically. It seems rather foolish *not* to use this knowledge. Doreian *et al.* (2005) proposed the idea of pre-specified blockmodeling as a way of incorporating this knowledge. This was facilitated by two mathematical theorems. Batagelj *et al.* (1992 a, b) proved that the most used types of equivalences, structural and regular, implied specific block types. If an unsigned network could be partitioned exactly in terms of structural equivalence, there were only two possible types of blocks – complete and null. For regular equivalence, they were null and one-covered (each row and each column in a block has at least one 1). They also expanded greatly the number of block types for inclusion in a blockmodeling analysis. Additionally, instead of using correlational and distance measures constructed from the network ties, blockmodels could be fitted directly by focusing on types of blocks and the distribution of ties in them. They ‘reversed’ the approach to using equivalence by defining new block types to create new types of equivalences. The positive and null blocks defined by Doreian & Mrvar (1996) were two new block types. In general, as new network domains are studied, the need for expanding the number of useful block types will grow for substantive reasons.

Pre-specified blockmodeling takes the form of identifying some - or all if we wish to be very ambitious - block types by location in a blockmodel. The structure theorems implicitly pre-specified the form of the blockmodel for signed networks. For a limited time, it worked well for the early collected signed networks. Pre-specification was used more successfully when using relaxed structural balance by Doreian & Mrvar (2014, 2015) as noted by Esmailian *et al.* (2014). Prota & Doreian (2016) used pre-specification to great effect for delineating the structure of rice trading (unsigned) networks in Vietnam. Despite these successes, pre-specification can be a double-edged sword, one to be used judiciously.

If substantive knowledge alone is used, there is no problem. Pre-specifying a blockmodel and fitting it to network data permits a test of the underlying substantive ideas. If the blockmodel fits there is some confirmation for these ideas. But if it does not fit, these ideas become suspect – assuming the data are good. But if empirical knowledge of the specific studied network is used too much, then there is the risk of ‘over-fitting’ a blockmodel and reporting only what one sees in the data. For this reason, the partition shown in Figure 4 tested nothing. While it is only an empirical description, it will be useful for formulating ideas about the temporal voting patterns of the Justices. Finding the right balance when using pre-specification will be difficult – but it must be identified.

### 3.4 Data issues

When structural balance was first proposed, and for several decades thereafter, the default mode for collecting social network data in human groups was a fixed choice design. This design restricts the number of responses to a small number of ties, most often 3. This was expanded to allowing ‘up to’ some specified number. While this is a modest improvement, the use of the fixed choice design remains very suspect from a measurement perspective despite its supposed convenience. Wasserman & Faust (1994) discuss this extensively about how this design introduced potentially severe measurement errors. This impacted the study of signed networks as well. Until recently, most of the analyses done with signed networks had, roughly, the same number of positive and negative ties. This had major consequences. The measures of imbalance and the use of the criterion function,  $C_f$ , defined above made sense if the number of negative and positive ties are the same. But what if they are not? If better network data collection methods are used or if signed graphs are studied in other empirical domains, the

number of positive and negative ties could differ, sometimes greatly. For example, in addition to producing the plot shown in Figure 3, Doreian & Mrvar (2016) found the number of positive ties far exceeded the number of negative ties. Hitherto, the line index of imbalance and the proportion of imbalanced triples had corresponded closely. Yet, in this network, they found that the line index of balance and the proportion of balanced triples did not correspond well: They pointed to different stories about changes in imbalance. Instead, the *number* of imbalanced triples corresponded well to the line index of imbalance. The presence of so many positive triples dramatically affected the classical measure of imbalance in signed networks, a problem that had not been recognized previously.

With the fixed choice design for collecting social network data having been abandoned largely – but not completely, alas – and with researchers considering other types of signed networks where the number of positive and negative ties differ considerably, serious problems have arisen. Particularly acute is the choice of  $\alpha$  in the criterion function for signed blockmodeling. The idea of using  $C_f$  with  $\alpha$  as a parameter was prompted by wanting to introduce some flexibility. It implied some consideration of which value of  $\alpha$  to use for a given network. But this was not explored in the literature. Yet, with the above considerations, using  $\alpha = 0.5$ , while seemingly reasonable, now seems quite arbitrary. Other values for the parameter merit attention. Examining the use of different values of  $\alpha$  has started. Initial experiments with changing  $\alpha$  have shown that the blockmodel partitions of signed networks can differ with the values of  $\alpha$  used as well as the line index measure of imbalance. The selection of  $\alpha$  in a principled fashion is a major unsolved problem. Until it is solved, a question mark will hang over using signed blockmodeling. Using the proportion of imbalanced triples as a measure can be questioned also. Solving the ‘alpha variation problem’ will not be easy.

In terms of data, further issues arise. One is that we are able now to collect valued signed data. This poses no problems for the Doreian-Mrvar algorithms as implemented in Pajek. Yet the ways in which the values of the signed ties are operationalized in different instruments may matter. This merits further attention also. A more troublesome issue is the presumption that the signs of ties are either positive or negative (or null) when measured. Cartwright & Harary (1970) raised the issue of considering ambivalent relations that are real but neither positive nor negative. Such ambivalence could be quite stable. Or it could point to the temporal instability of ties that could be positive at one point in time but negative at another. Incorporating ambivalence into the study of signed relations is another problem for both substantive and methodological reasons. Certainly, the current signed blockmodeling approach of Doreian and Mrvar cannot handle ambivalence. Ignoring this is no longer an option – yet another difficult problem to solve. This problem is addressed, but only partially, in Mendonça *et al.* (2015) in their study of voting in the European Parliament with abstentions from voting. As they note, the meaning of ‘abstaining’ is not clear. Their response took the form of examining alternative ways of coding this tie. In human relations between individuals this approach may not work so well as ambivalence implies both a positive and a negative tie between actors – or a qualitatively different type of tie.

All the problems outlined in the previous four sub-sections appear to imply a serious consideration of the ways in which data are collected. On the one hand, implicit in the foregoing is a demand for obtaining more and higher quality data. Yet, collecting data can be expensive and there are considerations regarding respondent fatigue in single instruments and, even more, in longitudinal studies. A very good resource for designing research to obtain useful data is Robins (2015). There, studies must be designed carefully from the outset to their conclusion. Given the difficulties of developing high quality data, it is not surprising that obtaining and using data electronically is so appealing. While such data can be obtained within well designed studies, convenience often dominates as the main criterion for obtaining data. When this happens most, if not all, of the considerations regarding substance described above

appear irrelevant. While we are clearly in an era of Big Data, I have major reservations about the adequacy of these data when substance is ignored. That data exist does not imply they are worth analyzing simply because they exist. Of course, this observation can be extended to small data also!

### 3.5 Algorithms

Assuming clean data can be collected for signed human relations, there are issues concerning the algorithms we use that must be addressed. The examples of the UNGA voting and Supreme Court voting described above suggest that, too often, our algorithms are just sledgehammers unfit for the subtleties of data in specific networks. The partitions established by Doreian & Mrvar (2015) were established after a painstaking examination of data once it was clear that well-known algorithms for partitioning such data failed. Of course, a painstaking look at the data may be necessary despite the risk of over-fitting the models to the data. Yet, it would be a major advance if efficient algorithms could be established for discerning the overall structure of such a network without the risk of over-fitting. Some research on this is under way but it is far from complete. One effort involves an ongoing collaboration between Traag, Doreian and Mrvar for a volume on partitioning networks to be published by Wiley. Clearly, Esmailian *et al.* (2014) have engaged on this issue as well with their consideration of relaxed structural balance. There is a well-known book review by Bonacich (2004) with the provocative title “The invasion of the physicists”. While he was careful to note that the ‘social’ social network community has something to learn from this invasion, it struck a chord. Clearly, members of the two fields do need to engage far more with each other.

The shift from partitioning signed networks using structural balance to using relaxed structural balance was compelling for substantive and empirical reasons. The move to partitioning two-mode signed networks was logical and very useful. Yet, there was a cost. For the criterion function defined above for classical structural balance, Doreian *et al.* (2006, p.305) established a theorem stating there would be a unique lowest value of the criterion function for structural balance that would occur either for one number,  $k$ , of clusters in the partition or a set of adjacent values of  $k$ . Loosely, the criterion had a U-shape with a unique minimum value for the criterion function.

For relaxed structural balance, Doreian & Mrvar (2009, p.5) proved that, if the number of clusters is denoted by  $k$ , the criterion function declines monotonically as  $k$  increases. For signed two-mode networks, with  $k_1$  and  $k_2$  the number of clusters of the rows and columns respectively, Mrvar & Doreian (2009, p.204) proved the criterion function declines monotonically with  $k_1$  for each value of  $k_2$  and monotonically with  $k_2$  for each value of  $k_1$ . The last two theorems imply that establishing partitions with a unique lowest value of the criterion function is not possible when the number of clusters is much lower than the number of vertices in the network. This ushers in judgment calls as to which partition, if any, is the best one to be selected – unless ways can be established to determine an optimum solution. Usually, the value of the criterion function declines faster for low values of  $k$ ,  $k_1$  and  $k_2$  than for higher values of these parameters. In addition, to using judgment, there is an issue of efficiency in choosing the grain of a partition (choice of  $k$ ). The more general problem is that if we think of the plot of the criterion function against  $k$  (for one-mode data), or against  $k_1$  and  $k_2$  (for two-mode data), in a three-dimensional plot, the shape of the plot is completely unknown. Worse, it may differ dramatically across the many different signed networks we could study. Therefore the partitioning of the UNGA data mentioned above was so difficult. Characterizing such plots in general may be intractable as a formal problem. The work of Mendonça *et al.* (2015) suggests one way forward.

#### IV SUMMARY AND SOME PROVISIONAL CONCLUSIONS

This reflection has provided a partial summary of one consistent approach to the study of signed relations that has been around for about 70 years. While it outlined some successes, clear failures were considered also. A closer look at these failures revealed a host of further major problems, both substantive and methodological. To build on the successes and, more consequentially, address the failures, these problems must be solved. Unless they are solved, the so-called balance theoretic approach will stall for there are looming potential dead ends. This adds urgency to pursuing the problems outlined here. No doubt, this must be done, especially as there are serious implications for developing adequate theories, designing both empirical and simulation studies of signed networks, collecting good data and employing well-designed algorithms for analyzing good data from different contexts.

At the risk of being viewed as a complete contrarian, I am not convinced of the value of focusing on the equilibrium states of signed networks. Given that signed networks move towards balance and away from balance, it seems highly unlikely that there will be long-term equilibria for them. Signed networks are highly dynamic. Equilibria appear to exist only in simulation studies.

As noted at the outset, there are multiple approaches to signed data. It may well be that ideas in some of these other approaches will be useful for solving some of the problems considered here. Clearly, Aref *et al.* (2016) provide such an example. It is possible, also, there are some ideas within the balance approach that will help solve some problems in these other approaches. See Mendonça *et al.* (2015) and Esmailian *et al.* (2014). More generally, sharing ideas across multiple approaches and between different disciplines seems the best way forward. It will be a massive quest but, I hope, not one that turns out to be quixotic. An academic friend, who changed sub-fields regularly, once remarked to me about research “When the easy problems have been solved, it is time to move on.” I disagree: Once the easy problems have been solved, research life gets far more interesting. There are some juicy and interesting problems to chew upon when studying signed networks. It will certainly be an exciting and fun ride for those pursuing it.

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