Computational & Mathematical Organization Theory, 10, 45–68, 2004

Identifying Linking-Pin Organizations in Inter-Organizational Networks

PATRICK DOREIAN
KAYO FUJIMOTO
Department of Sociology, University of Pittsburgh, Pittsburgh, PA 15260, USA
email: pitpat@pitt.edu
email: fujimoto@pitt.edu

Abstract

Within the large literature on inter-organizational networks, there has been some discussion of linking-pin organizations and the role they play in integrating these networks. Based on this verbal specification of linking-pin organizations, we construct operational criteria and empirical methods for identifying these structurally important organizations in potentially large and complex inter-organizational networks. These methods are based on ideas drawn from blockmodeling, structural holes, centrality and centralization of networks, and identifying cut-points in networks. These methods are applied to a constructed example and then to real empirical inter-organizational networks. Implications and contrasts with other methods are discussed, together with some open problems.

Keywords: cut points, interorganizational networks, centrality, structural holes, organizations

Organizations form relations with other organizations in efforts to obtain many of the resources they need to operate. Ties formed to help generate the flow of resources into an organization carry with them the risk of losing some autonomy to organizations from which the resources flow. This suggests that organizations are constrained to limit the number of those ties while actively forming relations with other organizations. A variety of substantive approaches all converge on the formation and structuring of interorganizational networks. Resource dependency theorists (notably, Pfeffer (1987), Pfeffer and Salancik (1978)) discuss the environments of organizations as conduits of resources and consider how, at the same time, these organizations form ties that constrain a focal organization. From an institutional perspective (see Meyer and Rowan (1991), DiMaggio and Powell (1991), Scott and Meyer (1991) and Doreian and Woodard (1999)) an emphasis is placed on institutional forces shaping the environments of organizations and the institutionalized forms adopted by organizations in those environments. Embeddedness (Granovetter, 1985) of agents, be they individuals or organizations, in structures of social relations affects the behavior of the agents and the institutionalized forms of the social structures. Institutions, interorganizational networks and organizations co-evolve in structured contexts.

Laumann et al. (1978) provide an early statement on interorganizational linkages in the context of community structure and discuss the implications of having organizations as nodes of 'ecological' networks. The ecological emphasis is present also in Astley and Fombrun (1978) with an emphasis on the evolving structure of interorganizational networks. More recent studies of collaboration (e.g. Powell et al. (1996)) and cooperation (e.g.

Gulati and Singh (1998) emphasize the dynamics of the coordination structures that form as organizations form relations in a dynamic context of other relations being formed. Powell and Smith-Doerr (1994) discuss networks as a form of governance with an emphasis on the processes of reciprocity, connectedness and embeddedness. All of these discussions emphasize the creation of coherent structures that form an ecological system of organizations. Given the presence of network structures, it seems useful to construct tools that delineate important structural features of these interorganizational networks and identify structurally significant organizations.

The set of all ties linking an organization to other organizations becomes a defining feature of that organization. They constitute its 'organization set' (Evan, 1966), an idea reiterated by Aldrich (1979) and by Whetten (1981). This shades naturally into the idea that the environment of an organization is made up of all the organizations with which the organization actually or potentially intereacts (Rogers, 1974). This formulation is implicit in the above cited discussions of interorganizational networks.

For a population of organizations, "an inter-organizational network consists of all of the organizations linked by a specific type of relation and is constructed by finding the ties between all the organizations (Aldrich, 1979:281)." This formulation can be extended to include multiple relevant relations between the organizations of a population. This is akin to DiMaggio's (1986) suggestion that a focus on the organizations of an institutionalized sector or domain (for some substantive interest) identifies an organizational field. While this is a general conceptual idea, Doreian and Woodard (1992) provide an empirically grounded method for identifying a population of organizations, as well as its organization field and a social network by an expanding selection method. Embedded within these organization fields are action sets of organizations as purposive interacting systems. Some organizations become distinctive over time in the inter-organizational networks (IONs) containing them. Our attention here is focused on linking-pin organizations as a subset of distinctive organizations.

1. Linking-Pin Organizations

Networks evolve through the operation of processes whereby ties are created, maintained or destroyed over time (Doreian and Stokman, 1997) and many structural forms can be reached. Linking-pin organizations are thought to have an integrative role in inter-organizational networks. For many populations of organizations, the network evolutionary processes generate a network structure with some denser patches of ties—viewed as subsystems—that are loosely coupled in some overall structure (Whetten, 1981) and linking-pin organizations perform a critical role in this loose coupling. Whetten views the ties coupling subsystems together as linking-pin relationships between a few representatives (of the subsystems). Put differently, "linking-pin organizations have extensive and overlapping ties to different parts of the network (Aldrich, 1979)." Throughout our discussion, we will use the term 'source network' for the original data matrix (or matrices) containing the relational ties.

According to Aldrich (1979:328–9), linking-pin organizations can perform a variety of functions. These include serving as communication channels between other organizations and providing general services linking organizations through the transfer of information, clients, personnel and other resources. Aldrich and Whetten (1981) argue that influence flows *between subsystems* through a small number of linking-pin organizations. Clearly, linking-pin organizations are structurally critical in their role of connecting inter-organizational networks. Given this, it seems important to establish methods for identifying these organizations in large complex networks.

A related discussion is provided by Burt's (1992) discussion of structural holes in networks of ties among social actors. He links his discussion to Granovetter's (1973) observations concerning 'the strength of weak ties' in connecting different parts of structures. In Burt's terms, a structural hole is a separation between non-redundant contacts and weak ties connect actors in separate 'clusters'. These ties are critical for the flow of information. For interorganizational networks, there is a strong parallel with the idea of linking-pin organizations being important for the flow of information. Burt (1992:27) is explicit that structural holes and weak ties appear to describe the same phenomenon but assigns greater causal priority to structural holes. This seems a misplaced emphasis as both are equally important. Burt notes that 'bridges' have two aspects "the chasm spanned and the chasm itself (1992:28)" and the structural hole argument is about spanning chasms in networks.

2. Structural Properties of Linking-Pin Organizations

Given the above broad characterization of 'linking-pin' organizations, we turn to specifying them and their properties more formally. We do this in order to establish procedures for identifying them in interorganizational networks. One ingredient is centrality (of organizations in networks). Aldrich (1979:327) argues that linking-pin organizations are central in IONs. This is consistent with the idea that centrality reflects the location[1] "of one unit relative to the others as determined by the number of relationships and the direct or indirect nature of those relations (Rogers (1974:488–9)." Restricting attention to the direct relations of an organization suggests the use of degree centrality (Freeman, 1979:219) as a possible feature of a linking-pin organization. However, if attention is focused on the indirect ties then some other feature of centrality is relevant. If linking-pin organizations integrate different parts of the network with the result that influence, information and other resources flow through them, then something like betweenness centrality (Freeman, 1979:222–3) seems more appropriate. Further, while the centrality of linking-pin organizations is critical, it seems that the 'integrating role' of a linking-pin organization will be reflected in the centralization of the network with its presence in the network.

Another ingredient for defining linking-pin organizations is found in Burt's (1992) use of the bridge imagery as a part of spanning chasms in networks. As defined below, each end of a bridge is a cut-point of a network: if either end of the bridge is removed, the network is disconnected. Linking-pin organizations, if they 'connect' networks may hold the networks together and their removal either disconnects the network or reduced its centralization. Now, in large complicated networks, with many ties, expecting to locate such disconnecting linking-pin organizations may be a too strong requirement. This brings us to the third ingredient in defining linking-pin organizations.

The discussion thus far is restricted to full inter-organizational networks and the central location of linking-pin organizations in those networks. It takes as a given that the different 'densely connected' parts of the network have been identified. This seems problematic especially in large networks. DiMaggio (1986) advocates a blockmodeling approach as an effective empirical approach to delineating network structure. The densely-connected subsystems will be seen as organizations that are structurally equivalent to each other. DiMaggio uses Aldrich's (1979) idea that linking-pin organizations are identified "as the only connection between different" positions.[2] This language suggests that linking-pin organizations will be distinctive in the blockmodel image of the source network. We suggest that blockmodeling not only identifies the clusters of (near) structurally equivalent actors, it will also identify the linking-pin organizations as *singletons in the blockmodel image* of the network. By this we mean that linking-pin organizations must be *structurally unique*. For an inter-organizational network, blockmodeling procedures produce clusters of organizations that are called positions. For an organization to be a linking-pin organization, we suggest that it must be in a cluster *by itself*.

This over-simplifies things in two ways. First, *both* the source network and the image of that network seem relevant for identifying linking-pin organizations in most practical situations. Second, not all singletons identified in a blockmodel image are linking-pin organizations. The Aldrich (1979) formulation of the nature of linking-pin organizations, reiterated by DiMaggio, suggests also that a linking-pin organization is a cut-vertex[3] in the image matrix (and may be a cut-vertex in the source network).

3. Some Formal Foundations

We draw on three areas for the foundations for the methods we propose using for identifying linking-pin organizations: (a) blockmodeling relational networks; (b) measures of centrality and centralization for networks; and (c) cut-points and cut-sets of graphs (as networks).

3.1. Blockmodeling Relational Networks

A graph is defined for a set of vertices (that, in this context, represent organizations) and relations defined over the vertices. Some relations are made up of ties that have no direction, for example, two organizations sharing one or more board directors. Such a relation is undirected and the ties are represented by *edges* where there is no direction. Other interorganizational relations do have a direction, for example purchasing products or referring clients. These directed ties are represented by *arcs* where the direction is important. More formally, a graph, G is an ordered triple $G = (V, E, A)$ where V is the set of vertices of the graph, E is the set of edges, and A is the set of arcs of G. Of course, E and A are pairwise disjoint sets and $L = A \cup E$ is the set of all relational ties. We let R be the social relation made up of the ties in L that are defined for pairs of elements in V. While the notational we use can be extended to multiple relations, we discuss a single relation here for simplicity.

The main objectives of blockmodeling, perhaps the most used social network analytic technique (Hummon and Carley, 1993), is the representation of role systems and the delineation of the fundamental structure of a network. For the latter the broad goal is to group together those organizations that belong together and can be 'shrunk' into a single vertex (called a position). This induces a partition of the elements of R into 'blocks' and the elements in the blocks are used to summarize the nature of the ties between positions. Put differently, clusters are units (actors) that share structural characteristics defined in terms of the relation R. (Each cluster forms a *position*.) The units within a cluster have the same or similar connection patterns to the rest of the network. More formally, the clusters form a partition $\mathbf{C} = \{C_1, C_2, \dots, C_k\}$. A clustering $\mathbf{C}$ partitions also the relation R into *blocks*

$$R(C_g, C_h) = R \cap C_g \times C_h$$

Each block[4] is defined in terms of units belonging to clusters C_g and C_h and consists of all lines from units in cluster C_g to units in cluster C_h. These lines are summarized in some fashion to express the block type.

3.1.1. Structural Equivalence. DiMaggio (1986) advocates the use of blockmodeling to study the structure of interoganizational networks and the early work on blockmodeling is based on the notion of structural equivalence. Lorrain and White (1971) introduced this idea and defined vertices as being equivalent if they are connected to the rest of the network in identical (the same) ways. More formally, a permutation $\varphi : V \to V$ is an *automorphism* of the relation R if and only if

$$\forall v_i, v_j \in V : (v_i R v_j \Rightarrow \varphi(v_i) R \varphi(v_j))$$

The units v_i and v_j are structurally equivalent, written as $v_i \equiv v_j$, if and only if the permutation $\pi = (v_i v_j)$ is an automorphism of the relation R (Borgatti and Everett, 1989).

In other words, v_i and v_j are structurally equivalent if and only if:

1. $v_i R v_j \Leftrightarrow v_j R v_i$ 3. $\forall v_m \in V \setminus \{v_i, v_j\} : (v_i R v_m \Leftrightarrow v_j R v_m)$
2. $v_i R v_j \Leftrightarrow v_j R v_j$ 4. $\forall v_m \in V \setminus \{v_i, v_j\} : (v_m R v_i \Leftrightarrow v_m R v_j)$

In words, for two structurally equivalent vertices, the first espression states that a tie from v_i to v_j can only exist if there is a reciprocal tie from v_j to v_i; the second expression states that if there is a reflexive tie (called a loop) at v_i then there has to be one at v_j; the third states that whenever there is a tie from v_i to some *other* vertex, denoted by v_m, there is also a tie from v_j to v_m; and the fourth expression states that if there is a tie from some (other) v_m to v_i, there must also be a tie from v_m to v_j. All must hold for two vertices to be structurally equivalent.

Letting r_{ij} denote the tie from v_i to v_j, the matrix expression of structural equivalence takes the form: $v_i \equiv v_j$ if and only if

1. $r_{ij} = r_{ji}$ 2. $\forall m \neq i, j : r_{im} = r_{jm}$
3. $r_{ii} = r_{jj}$ 4. $\forall m \neq i, j : r_{mi} = r_{mj}$

A blockmodel is a structure obtained by identifying all units from the same cluster of the clustering C and can be presented by a relational matrix, called an image matrix. The vertices in the reduced graph represent the positions, each of them defined by a cluster of units, and arcs represent the blocks[5] in the form of a summary of the ties in each block.

3.1.2. Establishing Blockmodels. There are two broad approaches to establishing blockmodels for social networks. These are the 'indirect approach' and the 'direct approach' Batagelj et al. (1992a). Under the indirect approach, a network of relations is converted to a matrix of (dis)similarities and then clustered by some clustering algorithm. One drawback with this approach is that care is needed in defining a measure of (dis)similarity that is compatible with the equivalence idea that is used. An equally important drawback is that different clustering algorithms introduce idiosyncratic features that can affect the partition identified.

The direct approach works with the data directly and uses some mathematical results that link an equivalence idea with the expected presence of blocks in a blockmodel. We consider again the notion of structural equivalence. If a blockmodel is exact (fully consistent with structural equivalence) then there are only two permitted block types: null blocks or complete blocks.[6] This insight is used to identify partitions that are as close as possible to being consistent with structural equivalence. To do this requires a measure of the discrepancy between an empirical blockmodel and a corresponding blockmodel that has only null blocks and complete blocks. A criterion function that measures departure of any partition from one that is consistent with structural equivalence can be specified. Let Θ denote a set of partitions that are all consistent with structural equivalence and let $\sim$ denote a blockmodel partition of the network. The properties of the desired criterion function, $P(\mathbf{C})$, are:

1. $P(\mathbf{C}) \geq 0$
2. $P(\mathbf{C}) = 0 \Leftrightarrow \sim \in \Theta$.

This first property states that the criterion function in non-negative and the second property states that the criterion function is zero if and only if the partition of the empirical blockmodel is exact (has only null and complete blocks).

The problem of establishing a partition of the network based on structural equivalence is a clustering problem that is solved by finding those partitions that have the smallest value of the criterion function. In short, it becomes an optimization problem where the value of the criterion function is minimized. The criterion function is non-negative (the first property) and if there are exact equivalences, then from the second property, the minimal value of $P(\mathbf{C})$ is 0. If Θ is empty, the optimization approach gives the solutions which differ least from some ideal model.

Given a clustering $\mathbf{C} = \{C_1, C_2, \ldots, C_k\}$, we let $\mathcal{B}(C_g, C_h)$ denote the set of all ideal blocks corresponding to block $R(C_g, C_h)$. (For structural equivalence, these blocks are either null or complete.) Then overall inconsistency of a clustering $\mathbf{C}$ (with an ideal clustering) can be expressed as

$$P(\mathbf{C}) = \sum_{C_g, C_h \in \mathbf{C}} \min_{B \in \mathcal{B}(C_g, C_h)} d(R(C_g, C_h), B)$$

where the term $d(R(C_g, C_h), B)$ measures the difference (number of inconsistencies) between the block $R(C_g, C_h)$ and a corresponding ideal block B. The term $d(R(C_g, C_h), B)$

can be expressed, for non-diagonal blocks,[7] as

$$d(R(C_g, C_h), B) = \sum_{v_i \in C_g, v_j \in C_h} \left| r_{v_i v_j} - b_{v_i v_j} \right|$$

In this expression, $r_{v_i v_j}$ is the observed tie and $b_{v_i v_j}$ is the corresponding value in a permitted[8] block. As noted above, there are only two kinds of block inconsistencies with structural equivalence: 0's in complete-blocks and 1's in null-blocks. The criterion function defined above treats these departures as equally consequential and simply counts them. They could be weighted differentially if one type of inconsistency is more important substantively than the other. (For a fuller statement of the problem, and a discussion of the local optimization method for minimizing the criterion function, see Batagelj et al. (1992a). These methods are used in establishing the blockmodels shown below. A block in the image is deemed a complete-block if more than 50 percent of the elements in the block are 1s.

In the main, blockmodeling has been used to establish partitions that are coarse grained in the sense of having as few clusters as possible. In our view, there are some limitations that follow from this presumption. Ideally, the units in a cluster are homogeneous but when there are relatively few clusters there is the risk of getting some clusters containing very dissimilar units. Burt (1976), rightly, is explicit about this and defines a 'residual' cluster, one that has little intrinsic meaning. Such a cluster may contain units that are truly distinct and ought to be identified as singletons each in their own cluster. In the following, we work with finer grained partitions than is usually the case with the intent of identifying linking-pin organizations as particular structurally unique units in inter-organizational networks.

3.2. Centrality and Centralization

In our discussion of 'centrality' in Section 2, we used the intuitive ideas of the number of ties an organization has as one concept of centrality and the extent to which an organization lies between pairs of other organizations as a second. We pursue these ideas here.[9] We use both the notion of a graph having ties (arcs and edges) and a relation (R) having elements that are the ties. We have R as a relation on (V, V) and let $v_i, v_j, v_m \in V$. R is made up of a set of elements $\{(v_i, v_j)\}$ and the transpose of R, denoted by R^{-1}, is made up of elements $\{(v_j, v_i)\}$ where $(v_j, v_i) \in R^{-1}$ if and only if $(v_i, v_j) \in R$. Throughout this section, we assume there are no loops in the network: $\forall v_i, (v_i, v_i) \notin R$. For a symmetric relation, R, $R = R^{-1}$. The symmetric closure of R is denoted by $\hat{R}$ and if R is symmetric, $R = R^{-1} = \hat{R}$.

3.2.1. Vertex Degree Measures. Let $S(v_i, R)$ be the set of elements, v_j, such that $(v_i, v_j) \in R$ and let $|S(v_i, R)|$ be its size. The out-degree centrality of v_i in R is $|S(v_i, R)|$ and the in-degree centrality of v_i in R is $|S(v_i, R^{-1})|$. In the imager of graphs, the out-centrality of v_i is the number of vertices v_m with an arc from v_i to v_m and the in-centrality of v_i is the number of vertices, v_m, with a tie from v_m to v_i. These measures are not normalized and do not involve the size of the network (n). However, to compare centrality measures across networks, it is necessary to control for the size of the network, n. In general, a simple normalized measure is obtained by dividing the centrality measure by its maximum value.

For a network with n vertices, the maximum value for both degree centrality measures is $n - 1$. The normalized out-degree and in-degree centrality scores are $|S(v_i, R)|/(n-1)$ and $|S(v_i, R^{-1})|/(n-1)$ respectively. If R is symmetric, then the centrality measure is $|S(v_i, R)|/(n-1)$ without ambiguity.

3.2.2. Vertex Betweenness Measures. Let $n(v_i, v_j)$ be the number of geodesics (shortest paths) from v_i to v_j and let $n(v_i, v_j; v_m)$ be the number of these geodesics that pass through v_m. The contribution of the pair of vertices, v_i and v_j to the betweenness centrality of v_m is given by $n(v_i, v_j; v_m)/n(v_i, v_j)$. The contribution of all pairs $\{(v_i, v_j)\}$ are summed and the betweenness centrality of v_m is $\Sigma_{v_i v_j} n(v_i, v_j; v_m)/n(v_i, v_j)$. Note that in this summation $v_i \neq v_j$. Again, the need to compare measures of vertex betweenness centrality across networks of different sizes implies the use a normalization of the measures. The maximum value of the simple measure of betweenness is $(n-1)(n-2)$ and $(\Sigma_{v_i, v_j} n(v_i, v_j; v_m)/n(v_i, v_j))/((n-1)(n-2)/2)$ is the normalized vertex betweenness centrality.

3.2.3. Network Centralization. In addition to examining the extent to which organizations are central, we need to examine the centralization of the network as a whole. Freeman (1979) suggested the following approach to defining centralization. For $\{c(v_i)\}$ as the set of centrality measures for $\{v_i\}$, let $c(p^*)$ be $\max\{c(v_i)\}$. For the network, $\Sigma_{v_i}(c(p^*) - c(v_i))$ is the sum of the differences in centralities for all vertices relative to the maximum centrality in the network. Denoting the maximum difference by M_c, a normalized centralization is $\Sigma_{v_i}(c(p^*) - c(v_i))/M_c$.

For degree centrality, with c_d denoting degree centrality, $M_c = (n^2 - 3n + 2)$ and the normalized degree centralization of a network is given by $\Sigma_{v_i}(c_d(p^*) - c_d(v_i))/(n^2 - 3n + 2)$. For betweenness centrality, with c_b denoting the vertex betweenness centrality measure, $M_c = (n^3 - 4n^2 + 5n - 2)$. The normalized betweenness centralization measure is $\Sigma_{v_i}(c_b(p^*) - c_b(v_i))/(n^3 - 4n^2 + 5n - 2)$.

3.2.4. Cut-vertices. If linking-pin organizations are important in connecting IONs, we can examine the extent to which the removal of a vertex affects the connectivity of the network. A directed path from v_i to v_j is a sequence of alternating vertices (that starts at v_i and finishes at v_j) and arcs where (i) each arc has its origin at the previous vertex and its end at the subsequent vertex, and (ii) no vertex or arc is repeated in the path. Intuitively, the direction of each arc is the same along the path. A semipath from v_i to v_j is an alternating sequence of vertices (that starts at v_i and finishes at v_j) and arcs where (i) successive pairs of vertices are connected from the first to the second, or from the second to the first and (ii) no vertex or arc is repeated in the semipath.[10] Intuitively, the direction of the arcs along the semipath need not be the same. Wasserman and Faust (1994:133) define four categories of connectedness for networks: (1) Weakly connected where all pairs of vertices, v_i and v_j, are joined by a semipath; (2) Unilaterally connected if each pair of vertices, v_i and v_j, are joined by a path in one direct or the other (but not both); (3) Strongly connected if $\forall v_i, v_j \in V$, there is at least one path from v_i to v_j and at least one path from v_j to v_i (without all the paths sharing the same vertices); and (4) Recursively connected where for a path from v_i to v_j there is a path from v_j to v_i that uses the same vertices in reverse order.

If a graph does not have one of the connectedness properties, a proper subgraph with the connectedness property is called a component of the graph. There can be weakly connected components, unilaterally connected components, strongly connected components and recursively connected components. A vertex, v_i, is a cut-vertex (relative to a connectedness type) if the network containing it has fewer components than the graph whose vertex set[11] is $V \backslash v_i$. While a full connectivity analysis would explore the transitions between graph types with the removal of a vertex, we focus on the qualitative structure of a network by constructing $\hat{R}$ and seeking single cut-vertices that disconnect the network. There is a natural dual concept of a cut-line as a tie whose removal increases the number of components (of a given type) or, in $\hat{R}$, disconnects the network. Such a line is called a bridge, a concept that features in Burt's (1992) discussion of structural holes.

4. Defining Linking-Pin Organizations as Cut-Vertices

For an ION, a *maximum* linking-pin organization is represented by a cut-vertex whose removal disconnects the ION. In empirical networks, such cut-vertices will be extremely rare. Such cut-vertices will have high betweenness centrality in the ION. It seems reasonable to entertain the idea that linking-pin organizations have high betweenness centrality in an ION. However, we suggest that the identification of linking-pin organizations is best done in the *image network* obtained via blockmodeling methods. The reason for this is that linking-pin organizations are structurally unique (in their location) in a network. This uniqueness goes beyond being extreme on some univariate distribution (e.g. centrality) describing the vertices. In turn, this implies that they are identified as *not* being equivalent to any other vertex. In addition, it seems important to distinguish cut-vertexes that disconnect only one other vertex[12] (or creates small components) and cut-vertexes that increase the number of large components.

We propose that a *necessary* (but not sufficient) condition for a vertex to represent a linking-pin organization is that it is a *singleton in a position of a blockmodel image*. Put differently, it is structurally unique and does not belong with other vertices in a position. A vertex not identified as a singleton in a position is *not* a linking-pin organization. The following arguments apply only for such vertices. We note that being a singleton in a position does *not* imply the vertex is a linking-pin vertex.

A *strong* linking-pin vertex is one whose removal disconnects the image network. Necessarily, such a vertex will have high betweenness centrality in the image. A singleton in the image that is not a cut-vertex is not a strong linking-pin vertex. Of course, there may be more than one strong linking-pin vertex in the image. Together, being a singleton in a cluster and being a cut-vertex in the image define a strong linking-pin vertex.

If there is no such cut-vertex, then we step back and suggest that a *weak* linking-pin vertex is one that does not disconnect the image but whose betweenness centrality in the image is the highest. Again, there may be multiple weak linking-pin vertices and we add the criterion that these weak linking-pin vertices must have the *highest* betweenness centrality measures in the image. Finally, as a secondary identifying criterion, we suggest that the network centralization of the image will be higher with the linking-pin vertex than the corresponding centralization of the image when the linking-pin vertex is removed. We turn now to consider some examples.

5. Examples

Having defined three kinds of linking-pin organizations (maximal, strong, weak) in inter-organizational networks and methods to identify them, we will use some examples to illustrate their use. While one example is artificial, the other examples are real, have increasing sizes, and show the identification of all types of linking-pin organizations. Throughout, the blockmodeling and graph drawing were done in Pajek (Batagelj and Mrvar, 2003) and for the centrality calculations we used Ucinet 6 (Borgatti et al., 2002).

5.1. An Artificial Example

The artificial example is extracted from the one used by DiMaggio (1986) and has an obvious structure: there are two maximally complete subnetworks (i.e. cliques) linked by a bridge between d and e. The network is shown in figure 1 and the permuted matrix of ties is shown in Table 1.

Both d and e are identified readily as cut-vertices in this network and, as such, are maximal linking-pin organizations. One position has $\{a, b, c\}$ and another has $\{f, g, h\}$. The blockmodel image of this network is shown in Table 1 with the partition imposed on the matrix. The partition is exact with regard to structural equivalence with d and e both cut-vertices in the image shown in figure 2. Of course, the example is an artificial one

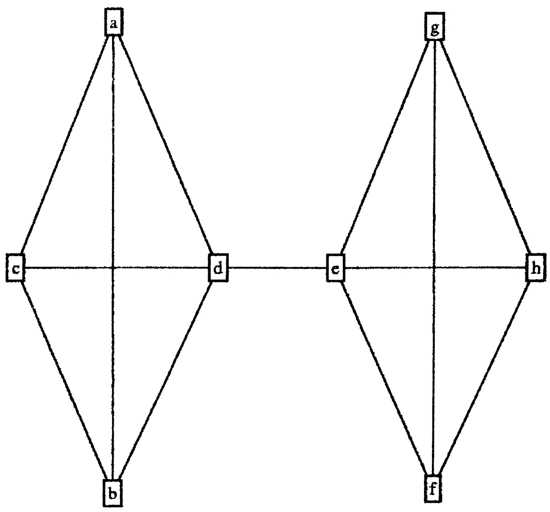

Figure 1. An artificial inter-organizational network.

Table 1. An Artificial inter-organizational networks with partition.

		1	2	3	4	5	6	7	8
a	1	0	1	1	1	0	0	0	0
b	2	1	0	1	1	0	0	0	0
c	3	1	1	0	1	0	0	0	0
d	4	1	1	1	0	1	0	0	0
e	5	0	0	0	1	0	1	1	1
f	6	0	0	0	0	1	0	1	1
g	7	0	0	0	0	1	1	0	1
h	8	0	0	0	0	1	1	1	0

Figure 2. An image of the artificial inter-organizational network.

and we suspect that, in general, there are few empirical networks where organizations are cut-vertices in the source graph. We note that the cut-vertices in figure 1 are identified as singletons in the exact blockmodel and are cut-vertices in figure 2. The centralization of the network in figure 1 is 0.49 and the centralization of the image in figure 2 is 0.44. When either *d* or *e* is removed from the image, the centralization of the remaining part of the image is 0. We note that the betweenness centrality of both *d* and *e* in the source graph is 0.57 (with the other vertices having zero betweenness centrality), a result suggesting that linking-pin organizations do have high betweenness centralities in the source network.

5.2. The Bandera Search and Rescue Mission (SAR)

Our second example comes from the Drabek et al. (1981) study of search and rescue (SAR) missions. One was in the Texas Hill country. Following an extended drought, steady rain began falling. While water was absorbed initially, soon most of it drained into creeks and rivers. The conditions for flash flooding were being created when seven inches of rain fell in little more than 12 hours and flood warnings were issued for three counties. One was Bandera County where eyewitnesses subsequently reported a 50 foot wall of water pounding through the county seat. The overall SAR mission was viewed by Drabek et al. as three separate SAR efforts in the three affected counties held together by state level and federal agencies, some of which had offices in the three affected counties. We focus here on the Bandera SAR mission. There is a mixture of organizational types in the emergent SAR network and they are numbered from 1 through 12. The US Army (1), stationed at Fort Sam Houston, is a national level organization. State level organizations include the Division of Disaster Services (2), the State Highway Patrol (3), the Texas National Guard (4), the Texas

State Parks and Wildlife Department (5), and the State Highway Department (6). The Red Cross (7) is a private organization. At the county level, there is the Emergency Management Services (8), the Bandera Sheriff (9), the Bandera Civil Defense (10), the Bandera Voluntary Fire Department (11) and the Pipe Creek Voluntary Fire Department (12).

5.2.1. Continuous Communication Only. In their narrative, Drabek et al. single out the (Texas) Highway Patrol as being particularly active with more linkages to other organizations in the SAR network than all other units. They noted also that the Sheriff's departments in each of the counties played 'key roles'. It seems reasonable to expect that these organizations would be linking-pin organizations. The data for the Bandera SAR are shown in Table 2 for continuous communication.[13] For our purposes here, the organizations to note are the two emphasized by the authors of the original study. The Highway Patrol is a cut-vertex in the source graph and is identified as a maximal linking-pin organization. The Bandera Sheriff is not such a linking-pin vertex.

The values of the criterion function (*cf*) for partitions into k clusters for $k = 2, 4, 3$ are, respectively, $cf = 32, 25, 21$. There are no unique clusters for these partitions and there are no singletons in any of the clusters (positions). However, for $k = 5$, there is a unique partition ($cf = 16$) with two singletons, the Highway Patrol and the Bandera Sheriff, each in their own cluster. This partition with the permuted matrix is shown in Table 2 and the image for this partition of the network is shown in figure 3 where positions labeled by the vertices appearing in them. For $k = 6$, there are three partitions of the network ($cf = 13$) and all have the same two singleton clusters identified for $k = 5$. We focus out attention on the partition in Table 2 and our criteria for identifying linking-pin organizations.

The summary information for this partition is shown in Table 3. The vertex representing the Highway Patrol forms a singleton cluster (and would be identified here as a linking-pin

Table 2. Permuted matrix of ties for Bandera SAR: Continuous communication.

Bandera (continuous)		3	9	1	5	8	11	12	2	4	6	7	10
Highway Patrol	3	0	1	1	1	1	0	0	1	1	1	0	0
Bandera Sheriff	9	1	0	1	1	1	1	0	0	0	0	0	0
USArmy	1	0	1	0	0	0	0	0	0	0	0	0	0
Wildlife	5	1	1	1	0	0	0	0	1	0	0	0	0
EMS	8	1	1	0	0	0	1	0	0	0	0	0	0
BC V Fire	11	0	1	0	0	0	0	0	0	0	0	0	0
Pk V Fire	12	0	0	0	0	1	0	0	0	0	0	0	0
DDIV EMS	2	1	0	0	0	0	0	0	0	0	0	0	0
National Guard	4	1	0	1	0	0	0	0	1	0	0	1	0
HwayDept	6	1	0	0	0	0	0	0	0	0	0	0	0
Red Cross	7	1	1	1	0	1	0	1	1	0	0	0	0
BC Civ Def	10	1	1	1	1	1	1	1	0	0	0	0	0

Table 3. Cut-vertex and betweenness centrality criteria: Bandera.

Organization	Cut-vertex	Centrality
Highway Patrol	Yes	0.42
Bandera Sheriff	No	0.17

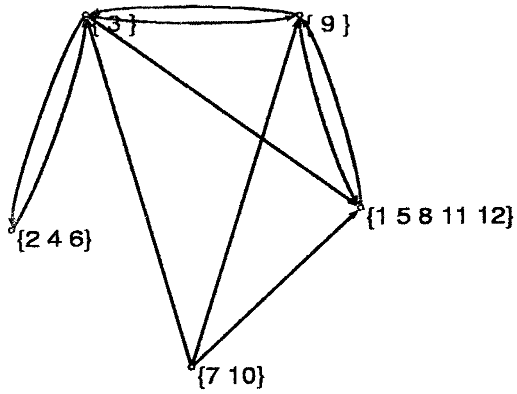

Figure 3. An image of the Bandera SAR network.

organization except that we know it is a maximal linking-pin organization). The vertex representing the Bandera Sheriff, while being a singleton, is not a cut-vertex. As shown in Table 3, these two vertices have normalized betweenness scores of 0.42 for the Highway Patrol and 0.17 for the Bandera Sheriff. The remaining clusters have zero betweenness scores. Use of these centrality measures identifies the Bandera Sheriff as a weak linking-pin organization. Clearly, networks can contain more than one linking-pin organization and there can be different types of linking-pin organizations in an ION. In the source network, the Highway Patrol has a betweenness centrality of 0.52 while the corresponding measure for the Bandera Sheriff is 0.22, again, suggestive of the idea that linking-pin organizations have high betweenness centrality in the source network.[14] The centralization of the source network is 0.91 and that of the image is 0.75. When the Highway Patrol is removed from the image, the resulting betweenness centralization is 0 and the corresponding score for the removal of the Sheriff is 0.67, results that are consistent with the former unit being a maximal (and strong) linking-pin organization and the latter a weak linking-pin organization.

5.2.2. All Communication. Drabek et al. provide data also on less frequent levels of communication and Table 4 provides the data when all communication levels are considered. We suspect that the introduction of weaker communication levels may act to mask the structure of the more critical continuous communication relation.

Table 4. Permuted matrix of ties for Bandera SAR: All communication.

Bandera (all)		3	1	5	11	12	2	4	6	7	8	9	10
Highway Patrol	3	0	1	1	1	0	1	1	1	1	1	1	0
USArmy	1	1	0	0	0	0	0	0	0	0	0	1	0
Wildlife	5	1	1	0	0	0	1	1	0	0	1	1	1
BC V Fire	11	1	0	0	0	0	0	0	0	0	0	1	0
Pk V Fire	12	0	0	0	0	0	0	0	0	0	1	0	1
DDIV EMS	2	1	1	1	0	0	0	0	1	1	0	0	0
National Guard	4	1	1	0	0	0	1	0	0	1	0	0	0
HwayDept	6	1	0	0	0	0	1	0	0	1	0	1	0
Red Cross	7	1	1	1	0	1	1	0	1	0	1	1	0
EMS	8	1	1	1	1	1	0	0	1	0	0	1	1
Bandera Sheriff	9	1	1	1	1	0	0	0	0	0	1	0	1
BC Civ Def	10	1	1	1	1	1	0	0	1	0	1	1	0

For $k = 2, 3$, there are no unique partitions and none have singleton clusters. (The values of the criterion function are $cf = 39, 34$ for these values of k.) For $k = 4$ there are three partitions where $cf = 27$ and all have the Highway Patrol as cut-vertex. As such it is a strong linking-pin organization. In the source network, its betweenness centrality is 0.25 (with the Bandera Sheriff coming next with a score of 0.09). In the image, it has the only non-zero betweenness centrality. The three partitions differ very little from each other and we show one of them in figure 4. For $k = 5$, there are four partitions ($cf = 23$) but only two of them have the Highway Patrol as a singleton. In those partitions it is not a cut-vertex but it does have the highest betweenness centrality of the positions. As such it is identified as a weak linking-pin organization.

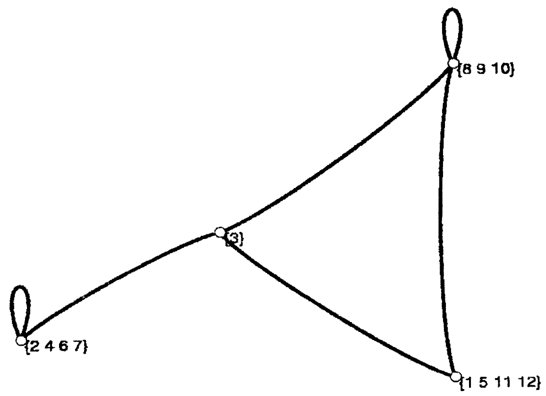

Figure 4. An image of the Bandera SAR network (all communications).

Table 5. Matrices of ties for the Kansas SAR: Continuous communication.

Partition with six clusters																					
Kansas SAR		1	5	6	7	2	9	3	4	14	16	17	8	12	13	10	15	18	19	11	20
Osage Sheriff	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1
HighwayP	5	1	0	1	1	0	0	1	1	1	1	0	0	0	1	0	0	0	0	0	0
Parks Resources	6	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0
GameFish	7	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Civil Def	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Army Corps	9	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0
Coroner	3	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Attorney	4	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Shawney	14	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1
LyndPolice	16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Red Cross	17	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
KansasDOT	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FrankCoAmb	12	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lee Rescue	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ArmyReserv	10	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BurlPolice	15	1	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1
Topeka FD	18	1	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
Carb Fire	19	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crable Amb	11	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Topeka RBW	20	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

5.3. *The Kansas SAR with Continuous Communication*

Another SAR considered by Drabek et al. (1981) occurred in Kansas on a lake where a tornado flipped a boat on a dinner and musical theater cruise on a lake. The identifying numbers of the organizations are given in Table 5 and used here in parentheses. The lake had been created by a dam built by the Army Corps of Engineers (9). The state park was administered by the Kansas State Parks and Resources Authority (6). Also having some jurisdiction was the State Game and Fish Commission (7). A variety of local police departments (15, 16), fire departments (18, 19), other county organizations (2, 3, 4), other state agency (8), another Army unit (10), some ambulance services (11, 12), the Red Cross (17) and some underwater rescue teams (13, 14) were involved. Data were collected from 20 responding organizations and we focus our attention on the continuous communication ties between them. The data are shown in Table 5.

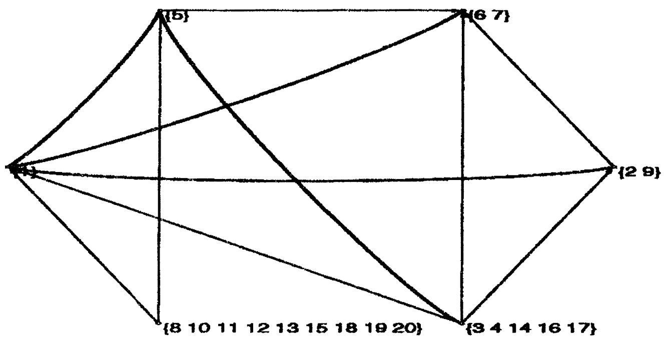

Figure 5. An image of the Kansas SAR network with six positions.

The Osage County Sheriff (1) took control of the SAR mission once his authority was assured by the County Attorney (4). He established his command post in a mobile communication unit of the Kansas State Highway Patrol (5). These two organizations were at the core of the whole SAR mission's network. Again, as was the case in Bandera, it seems reasonable that these two units—a local Sheriff and the Highway Patrol—are linking-pin organizations for the ION. Neither is a cut-vertex in the source network and there are no maximal linking-pin organizations in this network. The centralization of the source network is 0.41. The highest betweenness centrality scores are: Highway Patrol (0.23), the Osage Sheriff (0.18) and Parks & Resources (0.10). We turn to the blockmodel images of this network.

There are no singletons is their own clusters for $k = 2, 3, 4, 5$. There is, however, a unique partition for $k = 5$ with $cf = 42$. There is also a unique optimal partition for $k = 6$ with $cf = 38$ where there are two singletons each in their own cluster. Both the Osage Sheriff and the Highway Patrol occupy those singleton clusters as shown in Table 5. The image of this blockmodel is shown in figure 5. Neither of the two singleton organizations is a cut-vertex in the image (for $k = 6$) and are not strong linking-pin organizations. However, they do have the highest betweenness centrality measures (0.35 for the Osage Sheriff and 0.22 for the Highway Patrol) in the image and are identified as weak linking-pin organizations (see Table 6). The centralization of the image is 0.60, a value that is larger than that of the source network. When the Osage Sheriff is removed from the image, the centralization drops to 0.42 and when the Highway Patrol is removed, the measure drops to 0.50. The narrative of Drabek et al. (1981) makes it clear that the SAR effort is controlled more by the Sheriff than the Highway Patrol. This is obscured somewhat in the source network while it is revealed in the blockmodel image.

Table 6. Cut-vertex and betweenness centrality criteria, Kansas.

Organization	Cut-vertex	Centrality
Highway Patrol	No	0.22
Osage Sheriff	No	0.35

For $k = 7$ there are two partitions[15] (with $cf = 34$) and both have the same two singleton clusters. Both the Osage Sheriff and the Highway Patrol are weak linking-pin organizations for $k = 7$. For $k = 8$, some problems appear. First, there are many partitions with the minimum value of the criterion function. Such instability suggests that there is little of value with these fine grain partitions. In all of them, the identified linking-pin organizations do remain singletons in clusters. However, there are other singleton clusters that are identified. Obviously, as the grain of the partition gets finer and finer, there is a greater likelihood that singletons will appear. This suggests that seeking finer-grained partitions than is usually the case can be pushed only so far. For this network (with $k = 8$), we have reached a practical limit with regard to fine-grained blockmodels.

5.4. A Social Service Agency Network

Our final example concerns a network of social service agencies providing services to children and youth in a county system in the U.S. A fuller description is provided by Doreian and Woodard (1999) where four relations were considered simultaneously. Here, we focus on one relation: agencies that provide the majority of referral in ties. While there are 70 organizations in the network, some organizations are distinctive. These include Children and Youth Services (CYS), the dominant agency that straddles both the mental health and criminal justice sectors. Given the focus on mental health agencies, two organizations—a Mental Health Office (MHO) and the Community Mental Health Center (CMHC)—stand out in that sector (and form its core when viewed separately). The County Assistance Office (CAO) is the primary organization in the Poverty/Social Welfare sector. There are no cut-vertices in the source graph. One conjecture is that some or all of these agencies are linking-pin organizations.

The (70×70) matrix was partitioned in terms of structural equivalence with an increasing number of clusters. We report the unique partition with 17 clusters.[16] The image of this network is shown in figure 6 with five singleton positions. Four of them are CYS, MHO, CMHC, and CAO. There is a fifth singleton, CACT, another organization in the poverty sector. These five singletons satisfy the necessary condition for a linking-pin organization.

When the source network is considered, the three units with the highest betweenness centralities are CYS (0.21), CAO (0.19), and MHO (0.12). The five singletons are identified by their label and the remaining positions are labeled p6, p7 etc. In the image, there are two cut-vertexes. One is CYS whose removal disconnects p14, a cluster of eight agencies of which seven come from the criminal justice sector. Their connection to the rest of the system is through CYS which, as noted above, is located in two sectors. The other cut-vertex is CAO: the removal if this vertex disconnects p7 which is made up of a cluster of other agencies serving the poor. Both CYS and CAO are strong linking-pin organizations.

In the image, the top four singleton clusters, in terms of betweenness centrality, are CAO (0.06), CYS (0.06), CMHC (0.04) and MHO (0.04). The overall centralization of the source network is 0.37 while the corresponding figure for the image is 0.11. The image is not a highly centralized network and the highest centrality scores are modest. While it seems reasonable to consider MHO and CMHC as weak linking-pin organizations, their betweenness measures seem too small for such a delineation.[17] The fifth singleton in a

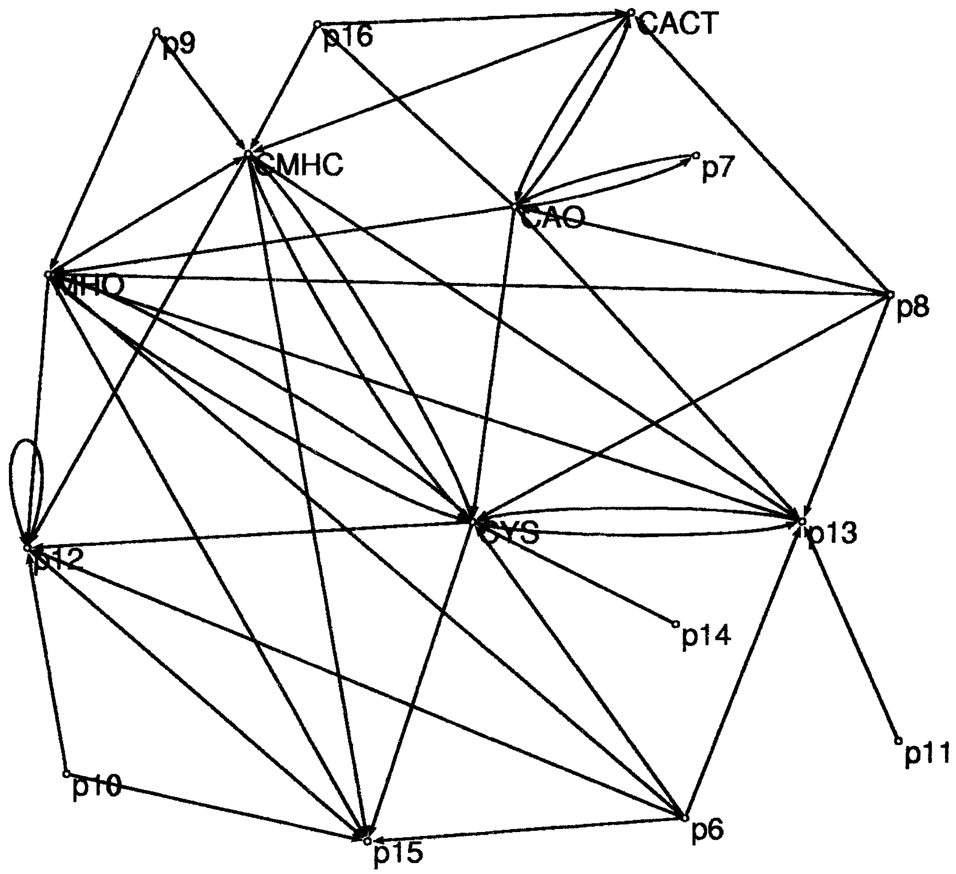

Figure 6. An image of the social service network.

cluster is CACT and its betweenness centrality is even lower (0.01) and there is a non-singleton cluster ranking higher than CACT on this measure. In short, there are just the two strong linking-pin organizations in this network.[18] This conclusion is tempered somewhat if attention is given to degree centrality. In the image, for example, CAO has an in-degree centrality score of 0.19 while CYS has a score of 0.44. However, the corresponding scores for CMHC and MHO and 0.31 and 0.38 suggesting that the linking role of these two agencies in the mental health sector is considerable when degree centrality is considered.

6. Structural Hole Analyses

In our initial discussion of linking-pin organizations, we made use of some of the arguments of Burt (1992) concerning structural holes as network features that can be spanned by organizations (to their benefit). It is instructive to consider some of Burt's indices concerning effective network size, efficiency and constraint as alternatives to using betweenness centrality.

For the source graph of the artificial organizational network, vertices *d* and *e* have the extreme values for the three indices generated during a structural hole analysis with the highest values for effective network size, highest values for efficiency and the lowest values for constraint. The same holds in the image network. Using structural hole analysis instead of betweenness centrality would have identified the same linking-pin organizations by type. For the Bandera network, with continuous communication, the organization with the highest effective network size, highest efficiency and lowest constraint is the Highway Patrol. This is true also for the image network. Again, we have the same linking-pin identification as with betweenness centrality. However, the next two organizations with the extreme values of the Burt indices are EMS and the Red Cross, neither of which were identified as linking-pin organizations because they are not singletons in a cluster. Put differently, they lack the structural uniqueness of a linking-pin organization in this network. Similar results hold for the analyses of the Bandera SAR when all communication levels are considered.

The social service delivery network is a much larger network with a more complex structure. Recall that: (i) CYS and CAO were identified as strong linking-pin organizations; (ii) both MHO and CMHC were identified as singletons in their own clusters but were not identified as weak linking-pin organizations given their low betweenness centrality in the image; and (iii) CACT, another singleton cluster did not even come close to being a weak linking-pin organization. When the structural hole measures are considered, CYS is identified as having the extreme scores. Next comes HMO followed by CAO, the organizations following these with the next extreme scores are all located inside clusters and do not satisfy the necessary condition for being a linking-pin organization. When the image network is considered, the extreme scores for the Burt indices reveal CYS, the first strong linking-pin organization identified earlier. The next pair of organizations that are identified are MHO and CMHC and would be identified as linking-pin organizations using a structural hole analysis. The next extreme organization is the CAO and would be identified as a linking-pin organization. Clearly, there is much in common in the results using betweenness centrality and structural hole analyses as the second identifying criterion of a linking-pin organization once the necessary condition has been satisfied. Yet there are differences and these need to be explored in future work.

7. Summary and Discussion

We have suggested an operationalization of the notion of a linking-pin organization and developed methods for identifying them in IONs. The key criteria involve uniqueness for a linking-pin organization. Most importantly, a necessary condition for a linking-pin organization is that it is a singleton in a cluster in a blockmodel image of the source network. Its pattern of ties to other organizations in the network is unlike the pattern of ties for all other organizations in the network. The second uniqueness criterion is that a linking-pin be a cut-vertex. If it is a cut-vertex in the source graph it is a maximal linking-pin organization.[19] It it is a cut-vertex in the image network, it is a strong linking-pin organization. These uniqueness criteria are unequivocal and differ from having uniqueness defined as the extreme score for some index.

If a vertex is not identified as a maximal or strong linking-pin organization, but it is still a singleton in a cluster, it may still have properties that make it structurally noteworthy in connecting a network. For a second criterion we used betweenness centrality but recognize that analyses based on structural hole considerations could have great merit. A weak linking-pin vertex is one with the extreme score with either set of measures. If there is more that one weak linking-pin organization, then they must have the most extreme scores on whatever set of measures is chosen for the second criterion. Identifying a weak linking-pin organization is a little fuzzy in operational terms because it is less explicit as a criterion. With maximal and strong linking-pin organizations, there is no ambiguity.

The method we propose couples together three extant tools: blockmodeling, identifying cut-vertices and using the extreme scores for indices that are readily computed for vertices. At this time, these are mere suggestions with limitations that require exploration. First, the blockmodeling as we have used it departs little from what Doreian et al. (2005) call conventional blockmodeling. Discussing one aspect of his structural hole approach, Burt (1992:42) notes that structural equivalence is too narrow as a definition of redundancy while regular equivalence is too broad. Batagelj et al. (1992a,b) point out that structural equivalence implies null and complete blocks while regular equivalence implies null and regular (1-covered) blocks. These two equivalence types from the point of departure for generalizing blockmodeling by incorporating many different types of blocks into a (generalized) blockmodel (Doreian et al. (1994, 2005). These more general types of blockmodels may be more appropriate for delineating linking-pin organizations in networks.

Second, given the primary rationale for blockmodeling is to discern the essential (simpler) structure of a network, there has been a bias towards using a few positions as possible. As noted earlier, the risk in doing this is the truly unique organizations may be clustered with other organizations in ways that are not helpful. In attempting to identify linking-pin organizations, we moved in the direction of obtaining finer grained partitions (using larger values of k). However, we have not explored how fine a partition is needed to identify linking-pin organizations. Really fine grained partitions necessarily will have singleton clusters and some of these will not be linking-pin organizations. This will vary from network to network. For the Kansas SAR, $k = 8$ already is too large. For the social service agency network we were able to go much higher in the range of k. We note that Doreian and Woodard (1999) used an even larger value of k. The sequencing of our examples suggest that larger networks pose more problems for the successful identification of linking-pin organizations.

Third, we have not provided an operational rule as to when a measure of betweenness centrality is 'high enough' in the image for a singleton cluster to be identified as a linking-pin organization. Instead, we specified that these organizations have to be extreme on whatever index is selected. No linking-pin organization, given it satisfies the necessary condition for being a linking-pin organization, can have a score that is lower than that of a cluster in the image. This use of the criterion may help with determining the grain of the blockmodel: once clusters as positions in the image have higher scores than singletons, the grain of the partition is too fine. In time, with more experience, we may be able to provide a precise threshold as a criterion. Of course, other measures of centrality may be more appropriate and useful. Certainly, there are many measures to consider but we do not review them here given the notion that, conceptually, betweenness centrality seems a better specification for looking at connectivity patterns. And, as our discussion in Section 6 suggests, we are open to the idea that structural hole analyses may be more fruitful for the second criterion for identifying linking-pin organizations.

Fourth, it will be necessary to know when to use single relations and when to use multiple relations in establishing the blockmodel images used to identify linking-pin organizations. It seems reasonable that when multiple relations are used, different organizations will be linking-pin organizations in different relations. The organizing framework of Baker and Faulkner (2002) suggests ways in which these methods can be used more generally in examining interorganizational networks. Finally, the Bandera example pointed to the need to consider carefully which form of an ION relation is considered if the arcs are valued.

While we have been concerned with the methodological issue of identifying linking-pin organizations, it is clear that there are theoretical and substantive issues that are raised by using these methods. Different types of linking-pin organizations may have different roles in IONs. More intriguing is the idea that different organizations are linking-pin organizations in different relations and that their functional roles may differ according to those relations. Another issue arises when the generation of linking-pin organizations is considered. As presented here, we have used the proposed method for identifying linking-pin organizations for a network as a given point in time. Yet, it is clear that the methods we suggest need to be employed in a longitudinal framework. Greve (2002) discusses clearly the nature of evolutionary network phenomena for interoganizational networks. In that context, linking-pin organizations do not just exist, they emerge in the context of network dynamics. Being able to identify the processes by which linking-pin organizations come into existence suggests the need to establsh substantive accounts for their emergence (together with the shaping of the structure of the ION as a whole). Additionally, linking-pin organizations represent only one type of 'structural uniqueness' and the other singletons in a blockmodel image of an ION will merit attention. A longer term goal is to establish a theory of structural uniqueness. There is much to be done and we hope that the methods proposed here are some useful first steps.

Acknowledgments

We appreciate greatly the extremely helpful comments of three reviewers, as well as the editors. These comments helped us improve our presentation substantially.

Notes

1. While Rogers uses the term 'position' for the 'location' of an organization in a network, this term is used in a technical sense in blockmodeling. We use location as a term for the set of ties an organization has with other organizations.
2. DiMaggio uses the term 'block' for 'position'. Again, block is a technical term, one that is reserved for parts of a blockmodel image and we use position as a term for a cluster of structurally equivalent organizations.
3. Another frequently used term is cut-point. This concept is defined below.
4. If $g = h$, the block $R(C_g, C_g)$ is called a *diagonal* block.
5. If the source network is symmetric then the image matrix will have only edges.

6. This differs slightly for diagonal blocks. The permitted diagonal blocks are complete with 0s in the diagonal of the block or null blocks with 1s in the diagonal. See Batagelj et al. (1992a) or Doreian et al. (1994).
7. This is modified slightly for diagonal blocks to ignore the diagonal elements in these blocks. (Also a diagonal block with 1s in the main diagonal and 0s elsewhere is consistent with structural equivalence.)
8. For structural equivalence, these can only be a null or a complete block. These are also called 'ideal' blocks.
9. Of course, there are many other centrality measures that have been defined. Flow measures, e.g. Stephenson and Zelen (1989) may be useful in this context. However we confine our attention primarily to betweenness centrality.
10. Strictly, a path satisfies the definition of a semipath but the reverse is not the case.
11. As a slight generalization, a set of q vertices whose removal increases the number of components—where the removal of fewer vertices does not increase the number of components—is a q-cut vertex set. Our focus is on cut-*vertices*.
12. Two such cases occurs in our last example (below).
13. Less frequent communication is ignored.
14. The next highest centrality score is 0.16 for National Guard.
15. These two partitions differ in the placement of only one vertex and have isomorphic images.
16. One of the clusters is quite large and ended up as an isolate in the image and is not included in figure 6. An even finergrained partition is suggested. Doreian and Woodard (1999) considered this in the context of analyzing four relations.
17. When CAO is removed, the centralization of the remaining part of the image remains 0.11. The corresponding figures for CYS, CMHC and MHO are, respectively, 0.12, 0.19 and 0.15. The changes for the two mental health agencies are the largest suggesting they do connect the network in a different way by making it less centralized with their presence.
18. Note that while p13 is a cut-vertex in the image, it is not a singleton cluster.
19. We conjecture that a cut-vertex in the source network will be a singleton in a cluster in the image network.

References

Aldrich, H.E. (1979), *Organizations and Environments*. Englewood Cliffs, NJ: Prentice-Hall.

Astley, W.G.R. and C.J. Fombrun (1987), "Organizational Communities: An Ecological Perspective," *Research in the Sociology of Organizations*, 5, 163–185.

Aldrich, H.E. and D.A. Whetten (1981), "Organization Sets, Action Sets and Networks: Making the Most of Simplicity," in P. Nystrom and W. Starbuck (Eds.), *Handbook of Organizational Design*, pp. 385–408.

Baker, W.E. and R.R. Faulkner (2002), "Interorganizational Networks," in J.A.C. Baum (Ed.), *The Blackwell Companion to Organizations*. Oxford: Blackwell, pp. 520–540

Batagelj, V. (1997), "Notes on Blockmodeling," *Social Networks*, 19, 143–155.

Batagelj, V. and A. Mrvar (2003), *Pajek: A Program for Large Network Analysis*. Lubljana: University of Ljubljana.

Batagelj, V., P. Doreian and A. Ferligoj (1992b), "An Optimizational Approach to Regular Equivalence," *Social Networks*, 14, 121–135.

Batagelj, V., A. Ferligoj and P. Doreian (1992a), "Direct and Indirect Methods for Structural Equivalence," *Social Networks*, 14, 63–90.

Borgatti S.P. and M.E. Everett (1989), "The Class of Regular Equivalences: Algebraic Structure and Computation," *Social Networks*, 11, 65–88.

Borgatti S.P., M.E. Everett and L.C. Freeman (2002), *Ucinet for Windows: Software for Social Network Analysis*. Harvard: Analytic Technologies.

Breiger, R.L., S.A. Boorman and P. Arabie (1975), "An algorithm for clustering relational data with applications to social network analysis and comparison to multidimensional scaling," *Journal of Mathematical Psychology*, 12, 328–383.

Burt, R.S. (1976), "Positions in networks," *Social Forces*, 55, 93–122.

Burt, R.S. (1992), *Structural Holes*. Cambridge: Harvard University Press.

DiMaggio, P.M. (1986), "Structural Analyses of Organizational Fields: A Blockmodel Approach," *Research in Organizational Behavior*, 8, 335–370.

DiMaggio, P.M. and B. Rowan (1991), "The Iron Cage Revisited: Institutional Isomorphism and Collective Rationality in Organizational Fields," in W.W. Powell and P.J. DiMaggio (Eds.), *The New Institutionalism in Organizational Analysis*. Chicago: University of Chicago Press, pp. 63–107.

Doreian, P., V. Batagelj and A. Ferligoj (1994), "Partitioning Networks Based on Generalized Concepts of Equivalence," *Journal of Mathematical Sociology*, 19, 1–27.

Doreian, P., V. Batagelj and A. Ferligoj (2005), *Generalized Blockmodeling*. New York: Cambridge University Press. (Forthcoming).

Doreian, P. and F.N. Stokman (1997), "The Dynamics and Evolution of Social Networks," in P. Doreian and F.N. Stokman (Eds.) *Evolution of Social Networks*, New York: Gordon and Breach, pp. 1–17.

Doreian, P. and K. Woodard (1992), "Fixed List Versus Snowball Selection of Social Networks," *Social Science Research*, 216, 216–233.

Doreian, P. and K. Woodard (1999), "Local and Global Institutional Processes," *Research in the Sociology of Organizations*, 16, 59–83.

Drabek, T.E., H.L. Tamminga, T.S. Kilijanek and C.R. Adams (1981), *Managing Multiorganizational Emergency Responses: Emergent Search and Rescue Networks in Natural Disaster and Remote Area Settings*. Boulder: Institute of Behavioral Science, University of Colorado.

Freeman, L.C. (1979), "Centrality in Social Networks: I. Conceptual Clarifications," *Social Networks*, 1, 215–239.

Granovetter, M. (1973), "The Strength of Weak Ties," *American Journal of Sociology*, 81, 1287–1303.

Granovetter, M. (1985), "Economic Action and Social Structure: The Problem of Embeddedness," *American Journal of Sociology*, 91(3), 481–510.

Greve, H.R. (2002), "Interorganizational Evolution," in J.A.C. Baum (Ed.), *The Blackwell Companion to Organizations*. Oxford: Blackwell, pp. 559–578.

Gulati, R. and H. Singh (1998), "The Architecture of Cooperation: Managing Coordination Costs and Appropriation Concerns in Strategic Alliances," *Administrative Science Quarterly*, 43, 781–814.

Hummon, N.P. and K. Carley (1993), "Social Networks as Normal Science," *Social Networks*, 15, 71–106.

Laumann, E.O., J. Galaskiewicz and P.V. Marsden (1978), "Community Structure and Interorganizational Relations," *Annual Review of Sociology*, 4, 455–484.

Lorrain, F. and H.C. White (1971), "Structural Equivalence of Individuals in Social Networks," *Journal of Mathematical Sociology*, 1, 49–80.

Meyer, J.W. and B. Rowan (1991), "Institutionized Organizations: Formal Structure as Myth and Ceremony," in W.W. Powell and P.J. DiMaggio (Eds.), *The New Institutionalism in Organizational Analysis*. Chicago: University of Chicago Press, pp. 41–62.

Pfeffer, J. (1987), "Bringing the Environment Back In: The Social Context of Business," in D.J. Teece (Ed.), *The Competitive Challenge: Strategies for Industrial Innovation and Renewal.* Grand Rapids: Harper and Row, pp. 119–135.

Pfeffer, J. and G. Salancik (1978), *The External Control of Organizations: A Resource Dependence Perspective*. New York: Harper and Row.

Powell, W.W., K.W. Koput and L. Smith-Doerr (1996), "Interorganizational Collaboration and the Locus of Innovation: Networks of Learning in Biotechnology," *Administrative Science Quarterly*, 41, 116–145.

Powell, W.W. and L. Smith-Doerr (1994), "Networks of Economic Life," in N. Smelser and R. Swedberg (Eds.), *Handbook of Economic Sociology*. New York: Russell Sage Foundation, pp. 368–402.

Rogers, D.L. (1974), "Sociometric Analysis of Interorganizational Relations," *Rural Sociology*, 39, 487–503.

Scott, W.R. and J.W. Meyer (1991), "The Organization of Societal Sectors: Propositions and Early Evidence," in W.W. Powell and P.J. DiMaggio (Eds.), *The New Institutionalism in Organizational Analysis*. Chicago: University of Chicago Press, pp. 108–140.

Stephenson, K. and M. Zelen, (1989), "Rethinking Centrality: Methods and Applications," *Social Networks*, 11, 1–37.

Wasserman, S. and K. Faust (1994), *Social Network Analysis*. New York: Cambridge University Press.

Whetten, D.A. (1981), "Interorganizational Relations: A Review of the Field," *Journal of Higher Education*, 52(1), 1–28.

White, D.R. and K.P. Reitz (1983), "Graph and Semigroup Homomorphisms on Networks of Relations," *Social Networks*, 5, 193–234.

Patrick Doreian is Professor and Chair of the Department at The University of Pittsburgh. He edits the *Journal of Mathematical Sociology* and is on the editorial board of *Social Networks*.

Kayo Fujimoto received her Ph.D. from The University of Pittsburgh in December 2003 and holds a fellowship from The Japanese Association for the Advancement of Science.